Pulling the Correct Lever: on Sticky Debt and the Common Factor Structure in Idiosyncratic Volatilities*

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Abstract

We show that time-varying financial leverage generates a common factor structure in firm-level idiosyncratic stock return volatilities (IVOL). A sufficient condition is to have sticky debt in a model where asset returns follow a simple linear single factor structure with constant volatility. Under reasonable parameter settings in a standard dynamic capital structure model, we numerically show that on average about 25% of the time variation in firm-level IVOL can be explained by a single factor. This proportion reduces to zero when using a purely equity-financed sample. We also show that the exposure to the common IVOL factor is negatively priced, even though IVOL is positively priced in the cross-section.

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1 Introduction

Recent empirical studies find that idiosyncratic stock return volatilities (IVOL) strongly co-move in the cross-section, and there is a common factor structure in IVOL (we denote this common factor as CIV). Duarte, Kamara, Siegel, and Sun (2014) and Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2016) both conclude that this common factor is priced, with Herskovic et al. (2016) arguing that this factor is driven by fundamental cash flow idiosyncratic volatility.[1] Our research investigates an alternative reason why IVOL co-move. We show theoretically and quantitatively that time-varying financial leverage generates co-movement in IVOL under very mild conditions, in which the residual stock returns can be uncorrelated and the fundamental cash flow idiosyncratic volatility can be a constant. We also show that financial leverage explains the negative cross-sectional pricing effect of the exposure to CIV shocks as documented in Herskovic et al. (2016).

We propose an equilibrium model in which firms' asset return processes satisfy a single factor CAPM. Each firm issues debt to maximize the equity value. The firms endogenously decide the capital structure choices by balancing the tax shield benefit against default cost. The firms can also restructure the capital structure as in Goldstein, Ju, and Leland (2001).

We show that financial leverage can generate a strong common factor structure in firm-level IVOL in a variant of the Goldstein et al. (2001) capital structure model where the firm's cash flow has a single factor structure. The Goldstein et al. (2001) upward refinancing framework permits closed-form expressions for equity values, and it also keeps financial leverage in a relative stable range (rather than vanishing) over a long simulation horizon. Under reasonable initial parameter settings, the cross-sectional average IVOL as a proxy for CIV explains more than 25% of the time variation in firm-level IVOL on average. The factor structure pattern in IVOL totally disappears if we use a purely equity-financed sample (no financial leverage). We conduct two further tests which show that financial leverage generates the co-movement in IVOL. First, we find that the average IVOL of the portfolios sorted by financial leverage are strongly correlated with each other except for the portfolio with lowest leverage. Second, in the market-level time-series regression, we find that the innovation in market average financial

[1] We use the term "IVOL" to specifically represent the equity return idiosyncratic volatility (as opposed to the fundamental cash flow idiosyncratic volatility).

leverage significantly determines the innovations in CIV.

Our findings are unaffected when we estimate IVOL from the simulation data using either the CAPM or five principal components of equity returns. The IVOL of the two models are highly correlated. The full panel correlation between the two IVOL estimates is 99%. Since principal component analysis (PCA) removes almost all the possible correlations in equity returns in the cross-section, we reach the same conclusion as Herskovic et al. (2016) to the extent that the IVOL co-movement does not arise from omitted factors in a linear factor model, where a factor is defined as a variable that drives covariance between returns. However, we show that the IVOL co-movement can arise from time-varying financial leverage, even when there is no factor structure in idiosyncratic volatilities at the asset level.

The above intuitions are not limited by the specific capital structure models we use in the paper. As argued in the main text, the appearance of common factor in the IVOL is a general result as long as the debt level is not proportional to the firm value. The essence of this intuition is that the equity value is not proportional to the firm value. Then the IVOL of the equity returns include firm asset value, and thus include the common factor of asset return process.

We also examine the cross-sectional pricing of CIV. Herskovic et al. (2016) find that the exposure to CIV shocks is negatively priced in the cross-section of stocks. We confirm this relation in the Goldstein et al. (2001) capital structure model. Firms in the lowest exposure to CIV quintile earn a value-weighted return that is 0.25% higher per month than firms in the top quintile. Furthermore, we propose a rationale to relate the negative pricing of CIV to financial leverage. Firms with higher financial leverage tend to suffer larger losses in equity value when there is a negative shock in the economy (i.e., a positive shock in CIV), so the firm's estimated exposure to CIV shocks will be negative. In addition, the loss in equity value further increases the leverage of such firms, and hence their expected return will increase. An analogous logic holds when there is a positive shock in the economy. As a result, the exposure to CIV shocks exhibits a negative relation with expected stock returns. We find supporting evidence from Fama-MacBeth regressions on simulated data in which financial leverage captures the negative pricing effect of exposure to CIV shocks.

We find a significant positive relation between lagged IVOL and the cross-section of stock returns. Stocks in the top IVOL quintile at the end of a month earn a value-weighted return that is 1.27% per month higher than stocks in the bottom quintile. In our analysis, IVOL is a positive monotonic transformation of the firm's leverage ratio. Provided the expected return on the firm's assets is positive, it is increasing in the firm's IVOL, with the leverage as a scale factor. Existing studies using empirical data to examine the relation between IVOL and expected returns find mixed results. On the one hand, Ang, Hodrick, Xing, and Zhang (2006) and a large number of follow-up studies find that stocks with high lagged IVOL exhibit low future returns (the so-called IVOL puzzle). On the other hand, the finding in our numerical analysis is more in line with Fu (2009) and Eiling (2013) who find a positive relation between conditional IVOL and expected returns.

We also identify a negative link between IVOL and the exposure to CIV shocks through financial leverage. This negative relation reconciles the positive pricing of IVOL and the negative pricing of the exposure to CIV shocks. For example, highly levered firms tend to have high IVOL but low (usually negative) exposure to CIV shocks, and both characteristics predict a high expected return in our study. Using empirical data, Herskovic et al. (2016) find a negative relation between IVOL and the exposure to CIV shocks, and are unable to reconcile the IVOL puzzle and the negative pricing of exposure to CIV shocks. Hence Herskovic et al. conclude that both anomalies co-exist.

Our study is related to the existing empirical literature that directly or indirectly studies how financial leverage affects the cross-sectional spread in expected stock returns. For example, Fama and French (1992) argue that size, book-to-market ratio and leverage are essentially different proxies that reflect information on risk and expected returns. Ang et al. (2006) find that leverage cannot explain the negative pricing effect of IVOL at the firm level. Choi and Richardson (2016) propose a method to compute the asset volatility and find that the presence of financial leverage creates a significant difference between the properties of equity and asset volatilities. Doshi, Jacobs, Kumar, and Rabinovitch (2016) de-lever equity returns based on the Merton (1974) and Leland and Toft (1996) capital structure models. Using the unlevered returns they find that the negative relation between stock returns and IVOL disappears. Doshi et al. (2016) focus on the role of financial leverage in explaining the cross-sectional equity returns anomalies, including size, value and IVOL. In contrast, our focus is the implication of financial leverage for the common factor structure in IVOL in the cross-section. We also use financial leverage to explain the pricing effects of IVOL and the exposure to CIV shocks.

Our study is also related to the literature on optimal capital structure models and its application to the study of IVOL. To the best of our knowledge, we are the first to apply capital structure models to study, in a cross-sectional setting, the role that financial leverage plays in the common factor structure of IVOL.

The paper is organized as follows. Section 2 proposes a simple model to illustrate how time-varying financial leverage generates the common factor structure in IVOL. Section 3 discusses the capital structure model, the setup of the simulation and the summary statistics. Section 4 analyses the behaviour of IVOL including its common factor structure, the pricing effect of the exposure to CIV shocks and lagged IVOL. Section 5 summarizes our conclusions and Section 6 discusses some possible extensions to our study.

2 A Theory of Leverage and Idiosyncratic Volatility

2.1 Model Setup: One-Factor APT (CAPM) Setup for Asset Return Processes

We consider a one-factor APT type model for firm assets. We assume that there are N firms, indexed by $i = 1, \ldots, N$, whose cum-dividend value processes follow

$$\frac{dV_i(t) + \delta_i(t)dt}{V_i(t)} = (\mu_i + \delta_i)dt + \sigma_i(\rho_i dW(t) + \sqrt{1-\rho_i^2}dZ_i(t)), i = 1, \ldots, N, \tag{1}$$

where $\delta_i(t) = \delta_i V_i(t)$ is the payout-value ratio, $\mu_i,\ \delta_i,\ \sigma_i,\ \rho_i$ are all positive constants, and $W(t),\ Z_i(t)$ are standard Brownian motions that are uncorrelated with each other, for all $i = 1, \ldots, N$. And there exists a risk-free bond with constant risk-free rate $r > 0$.

To make the model more concrete, we assume this factor is the market return. Define

$$V_m(t) \equiv \sum_i V_i(t),\ \delta_m(t) \equiv \sum_i \delta_i(t),\ \omega_i^A(t) \equiv \frac{V_i(t)}{V_m(t)}. \tag{2}$$

We assume APT holds such that the market return is only a function of the common shocks

$dW(t)$,

$$\frac{dV_m(t) + \delta_m(t)dt}{V_m(t)} = r_m^A(t)dt + \sigma_m^A(t)dW(t),$$

where we assume Law of Large Number (LLN) holds so the idiosyncratic volatility is diversified away in the market portfolio, and

$$r_m^A(t) \equiv \sum_i \omega_i^A(t)(\mu_i + \delta_i), \;\; \sigma_m^A(t) \equiv \sum_i \omega_i^A(t)\sigma_i\rho_i. \tag{3}$$

To make sure that the expected market return equals the risk-free rate under the risk-neutral measure defined below, we define the (time-varying) market price of risk as

$$\theta_m^A(t) \equiv \frac{r_m^A(t) - r}{\sigma_m^A(t)}. \tag{4}$$

Thus one can define a new risk-neutral measure generated by the Brownian motion

$$d\tilde{W}(t) \equiv dW(t) + \theta_m^A(t)dt, \tag{5}$$

so that the market return is of the form

$$\frac{dV_m(t) + \delta_m(t)dt}{V_m(t)} = rdt + \sigma_m^A(t)d\tilde{W}(t). \tag{6}$$

For firm i, we define the price of the idiosyncratic risk of firm Z_i as θ_i^A. The Brownian motion under the risk-neutral measure for individual firm can then be written as

$$d\tilde{Z}_i(t) \equiv dZ_i(t) + \theta_i^A dt. \tag{7}$$

As we will work on the risk neutral measure later on, we want to make sure that the expected return of each individual firm equals r under this new risk-neutral measure as well,

$$\frac{dV_i(t) + \delta_i(t)dt}{V_i(t)} = rdt + \sigma_i\rho_i d\tilde{W}(t) + \sigma_i\sqrt{1-\rho_i^2}d\tilde{Z}_i(t). \tag{8}$$

It follows that

$$\frac{dV_i(t)+\delta_i(t)dt}{V_i(t)} = \left(r+\sigma_i\rho_i\theta_m^A+\sigma_i\sqrt{1-\rho_i^2}\theta_i^A\right)dt+\sigma_i\left(\rho_i dW(t)+\sqrt{1-\rho_i^2}dZ_i(t)\right).$$

In other words, θ_i must satisfy

$$\mu_i+\delta_i = r+\sigma_i\rho_i\theta_m^A+\sigma_i\sqrt{1-\rho_i^2}\theta_i^A. \tag{9}$$

As pointed out in Cochrane (2005), individual θ_i^A can be anything for any finite set of securities. In other words, idiosyncratic risk can be priced under a pure APT model. For our purpose. we will assume that at the firm level, the APT holds exactly in a single factor (CAPM) model, i.e.,

$$\theta_i^A = 0 \quad \forall i. \tag{10}$$

Hence, idiosyncratic risk is not priced at asset level, (but idiosyncratic volatility can still be priced). Under this assumption, expected asset returns are given by

$$\mu_i+\delta_i = r+\sigma_i\rho_i\theta_m^A.$$

Define the asset return beta as

$$\beta_i^A \equiv \frac{\mathrm{Cov}\left(\frac{dV_i(t)+\delta_i(t)dt}{V_i(t)},\frac{dV_m(t)+\delta_m(t)dt}{V_m(t)}\right)}{(\sigma_m^A)^2}, \tag{11}$$

then the asset returns follow a CAPM where

$$\mu_i+\delta_i = r+\beta_i^A(r_m^A-r). \tag{12}$$

And the Brownian motions under the risk-neutral and objective probability measure satisfy

$$d\tilde{W}(t) = dW(t)+\theta_m^A dt,$$

$$d\tilde{Z}_i(t) = dZ_i(t).$$

2.2 Optimal Capital Structure for A Single Firm: Static Capital Structure

With the above setup, we now proceed to derive the optimal capital structure of a single firm. The model mainly follows Goldstein et al. (2001), so we will skip most of the proofs for the results in this section. Given the setup in the previous section, the cum-dividend expected return of any firm should be the risk-free rate r under the risk-neutral measure.,

$$\frac{dV_i(t) + \delta_i(t)dt}{V_i(t)} = rdt + \sigma_i \left(\rho_i d\tilde{W}(t) + \sqrt{1-\rho_i^2} d\tilde{Z}_i(t) \right). \tag{13}$$

To simplify the notation, we define for the firm i from now on in this section we will ignore the subscript i):

$$\begin{aligned} d\tilde{z}(t) &\equiv \rho_i d\tilde{W}(t) + \sqrt{1-\rho_i^2} d\tilde{Z}_i(t), \\ \nu &\equiv r - \delta_i, \\ \sigma &\equiv \sigma_i. \end{aligned}$$

It follows that the asset process under the risk-neutral measure follows,

$$\frac{dV}{V} = \nu dt + \sigma d\tilde{z}(t), \tag{14}$$

with a total payout process that is proportional to the current asset value, $\delta(t) = \delta V(t)$.

We start by considering the case in which the firm decides the optimal capital structure only once in the beginning, and will study the dynamic capital structure later. As shown in Goldstein et al. (2001), the optimal capital structure (coupon C) is given by:

$$C^* = \frac{rV_0}{\lambda} \left[\left(\frac{1}{1+x} \right) \left(\frac{A}{A+B} \right) \right]^{\frac{1}{x}}, \tag{15}$$

where

$$x = \frac{1}{\sigma^2}\left[\left(\nu - \frac{\sigma^2}{2}\right) + \sqrt{\left(\nu - \frac{\sigma^2}{2}\right)^2 + 2r\sigma^2}\right] > 0,$$

$$\lambda = \frac{x}{1+x},$$

$$A = (1-q)(1-\tau_i) - (1-\tau_{eff}),$$

$$B = \lambda(1-\tau_{eff})\left(1-(1-q)(1-\alpha)\right).$$

For a given coupon C, the equity value at any point after the debt issuance and before any default is given by:

$$E(C,V;V_B) = E_{solv} = (1-\tau_{eff})\left[V - V_B\left(\frac{V}{V_B}\right)^{-x} - \frac{C}{r}\left(1-\left(\frac{V}{V_B}\right)^{-x}\right)\right],$$

where

$$V_B = \frac{x}{1+x}\frac{C}{r} \equiv \lambda\frac{C}{r}. \tag{16}$$

2.3 Equity Returns Under Static Capital Structure

We now discuss the cross-sectional equity returns under the above static capital structure model. Using the value for V_B, we can write all the equity values in terms of optimal coupons

$$E(V,C^*) = (1-\tau_{eff})\left[V + G\left(\frac{V}{V_0}\right)^{-x} - \frac{C^*}{r}\right], \tag{17}$$

where

$$G = \frac{V_0}{x}\left[\left(\frac{1}{1+x}\right)\left(\frac{A}{A+B}\right)\right]^{1+\frac{1}{x}}.$$

Here the three terms between brackets in Equation (17) have clear economic meaning. The first term is the total value of the assets. The last term is the deduction of a risk-free debt. The second term is from the option value of the defaulting for the equity holder. In the following we use the above result to examine the properties of IVOL.

2.3.1 A simplified version

We start by only considering the first and the third terms. Namely we assume the debt is risk-free and ignore the option value for now. We consider this case to illustrate the intuition and will study the full case in the next subsection. Denote this equity value as E^s (superscript "s" means simple),

$$E^s(V) \equiv (1 - \tau_{eff})(V - C/r) \equiv (1 - \tau_{eff})(V - D^s), \tag{18}$$

where

$$D^s \equiv C/r, \tag{19}$$

is the face value of the consol bond.

The after-tax instantaneous dividend payoffs to the equity holders, denoted as $\Delta(t)$, equal $(1 - \tau_{eff})(\delta(t) - C)dt$. So the return on the equity value is given by

$$(1 - \tau_{eff})(dV + (\delta(t) - C)dt) = (1 - \tau_{eff})\left([V(\mu + \delta) - D^s r]\,dt + V\sigma dz\right),$$

where (we recover the subscript here)

$$dz_i(t) \equiv \rho_i dW(t) + \sqrt{1 - \rho_i^2}dZ_i(t). \tag{20}$$

The equity return process is given by

$$\frac{dE_i^s(t) + \Delta_i(t)dt}{E_i^s(t)} \equiv r_i^s dt + \sigma_i^s \left(\rho_i dW(t) + \sqrt{1 - \rho_i^2}dZ_i(t)\right)$$

where

$$\begin{aligned} r_i^s &= \frac{V_i(t)}{V_i(t) - D_i^s}(\mu_i + \delta_i) - \frac{D_i^s}{V_i(t) - D_i^s}r, \\ \sigma_i^s &= \frac{V_i(t)}{V_i(t) - D_i^s}\sigma_i. \end{aligned}$$

There are two sources of shocks to the equity returns; a systematic one associated with $W(t)$

and an idiosyncratic one associated with $Z_i(t)$. The IVOL of the firm is given by:

$$IVOL_i^s(t) = \sigma_i^s \sqrt{1-\rho_i^2} = \frac{V_i(t)}{V_i(t) - D_i^s}\sigma_i \sqrt{1-\rho_i^2}.$$

From here we can see the potential appearance of common factors in the IVOL because of the term $\frac{V_i(t)}{V_i(t)-D_i^s}$. Recall that $V_i(t)$ contains the history of common shocks $W(t)$ and it is a function of $V(t)$ by our setup. D_i^s is fixed under the static capital structure setting we use in this section, and will not change between two consecutive restructuring epochs. We can contrast this with the situation that D_i^s is proportional to $V_i(t)$, then the coefficient of the idiosyncratic risk becomes a constant, which would result in a constant idiosyncratic volatility. The key insight of our model is that as long as the leverage is not proportional to the value of the firm, the idiosyncratic volatility of the equity returns exhibit common variations.

Another insight from the above representation is that the common variation does not stem from omitted risk factors. All assets are driven by a single common factor which is correctly accounted for when computing idiosyncratic volatilities. The idiosyncratic shocks to firm asset values are completely independent across firms. The common movement in IVOL shows up as a multiplicative factor for the firm-level asset idiosyncratic volatility. Thus using additive econometric techniques such as principal component analysis (PCA) will not allow to completely capture this common variation.

We can calculate directly the common factor defined in the empirical literature in this simple case. If all firms have the same parameters, then the common idiosyncratic volatility factor CIV as defined in Herskovic et al. (2016) equals a constant times the average leverage in the economy,

$$CIV = \frac{1}{N}\sum_i \sigma_i^s \sqrt{1-\rho_i^2} = \sigma\sqrt{1-\rho^2}\frac{1}{N}\sum_i \frac{V_i(t)}{V_i(t) - D_i^s}.$$

If in addition $\rho = 0$, i.e., there is no systematic risk, then the following holds.

Proposition 1. *If all firms have identical parameters and there is no systematic risk, then the* *correlation between firm-level IVOL and the CIV factor equals*

$$Cov\left(\frac{V_i(t)}{V_i(t)-D_i^s}\sigma, \sigma\frac{1}{N}\sum_i\frac{V_i(t)}{V_i(t)-D_i^s}\right) = \sigma^2\frac{1}{N}\left(\frac{V_i(t)}{V_i(t)-D_i^s}\right)^2.$$

Note that if $\rho \neq 0$, then $V_i(t)$ and $V_j(t)$ for $i \neq j$ are not independent as they both depend on the cumulative common shock $W(t)$.

We can show that CAPM still holds for equity returns in this simple case. Furthermore, the IVOL is closely related to the equity beta. Define the market portfolio of equity as

$$E_m^s(t) \equiv \sum_i E_i^s(t) = (1-\tau_{eff})\sum_i(V_i(t)-D_i^s),$$

specify the relative weight of firm i as

$$w_i^s(t) \equiv \frac{E_i^s(t)}{E_m^s(t)} = \frac{V_i(t)-D_i^s}{\sum_i(V_i(t)-D_i^s)},$$

and define the market payout ratio as

$$\Delta_m(t) \equiv (1-\tau_{eff})\sum_i(\delta_i(t)-D_i^s r).$$

Then the market portfolio evolves as:

$$\frac{dE_m^s(t)+\Delta_m(t)dt}{E_m^s(t)} = r_m^s(t)dt + \sigma_m^s(t)dW(t),$$

where

$$\begin{aligned}
r_m^s(t) &\equiv \sum_i w_i^s\left(\frac{V_i(t)}{V_i(t)-D_i^s}(\mu_i+\delta_i) - \frac{D_i^s}{V_i(t)-D_i^s}r\right), \\
&= r + \frac{V_m(t)}{E_m^s(t)}(r_m^A - r), \\
\sigma_m^s(t) &\equiv \sum_i w_i^s\frac{V_i(t)}{V_i(t)-D_i^s}\sigma_i\rho_i, \\
&= \frac{V_m(t)}{E_m^s(t)}\sigma_m^A.
\end{aligned}$$

The covariance between individual equity returns and the market equity returns is given by

$$\text{Cov}\left(\frac{dE_i^s(t)+\Delta_i(t)dt}{E_i^s(t)}, \frac{dE_m^s(t)+\Delta_m(t)dt}{E_m^s(t)}\right) = \frac{V_i(t)}{V_i(t)-D_i^s}\sigma_i\rho_i\sigma_m^s(t),$$

Then the individual firm's equity beta is given by:

$$\begin{aligned}\beta_i^s(t) &= \frac{\text{Cov}\left(\frac{dE_i^s(t)+\Delta_i(t)dt}{E_i^s(t)}, \frac{dE_m^s(t)+\Delta_m(t)dt}{E_m^s(t)}\right)}{(\sigma_m^s(t))^2}\\ &= \frac{V_i(t)}{V_i(t)-D_i^s}\frac{\sigma_i\rho_i}{\sigma_m^s(t)}\end{aligned}$$

As a result, the expected stock return for firm i is:

$$r_i^s(t) = r + \beta_i^s(t)(r_m^s(t)-r).$$

Note the appearance of $\frac{V_i(t)}{V_i(t)-D_i^s}$ in both IVOL and the equity beta of the firm. The following cross-sectional result is immediate.

Proposition 2. *If all the firms have identical parameters, then the cross-sectional correlation between IVOL and the equity beta is positive.*

If the economy does poorly (well), then all firms will tend to become more (less) levered. Hence, if the economy does poorly (well), then IVOL will tend to increase (decrease) for all firms, and equity betas of relatively more-levered firms will tend to increase (decrease), while the equity betas of relatively less-levered firms will tend to decrease (increase). These effects are stronger the more the firm's leverage differs from the market average leverage.

2.3.2 Full Version

We now study the full equity value in (17), including the nonlinear term V^{-x}. The basic intuition on the common variation in the IVOL still holds. Nevertheless the relation between IVOL and equity beta is not as clear-cut. The main steps are the same as above, so we simply state the results without proof.

Note that the value process $V_i(t)$ and the constants G_i and C_i are all linear functions of the initial firm value $V_i(0)$. So to simplify the notation, in the following we scale them by the initial firm value $V_i(0)$. In other words, the equity value is given by

$$E_i(V_i, C_i^*, V_i(0)) = (1 - \tau_{eff})V_i(0)(V_i(t) + G_i V_i(t)^{-x_i} - D_i^s),$$

where D_i^s is defined as above. So the equity value process follows

$$dE_i(t) = (1 - \tau_{eff})V_i(0)[dV_i(t) + G_i(-x_i)V_i(t)^{-x_i-1}dV_i(t) + \frac{1}{2}G_i(-x_i)(-x_i - 1)V_i(t)^{-x_i-2}d[V_i(t), V_i(t)]].$$

The return process of the equity is then given by

$$\frac{dE_i(t) + \Delta_i(t)dt}{E_i(t)} \equiv r_i^E(t)dt + \sigma_i^E(t)(\rho_i dW(t) + \sqrt{1 - \rho_i^2}dZ_i(t)),$$

where

$$\Delta_i(t) = \delta_i V_i(t) - D_i(t)r.$$

The expected return and volatility are then given by

$$r_i^E(t) = r + \frac{V_i(t)}{V_i(t) + G_i V_i(t)^{-x_i} - D_i^s}(\mu_i + \delta_i(t) - r) + \frac{G_i V_i(t)^{-x_i}}{V_i(t) + G_i V_i(t)^{-x_i} - D_i^s}\left(-x_i\mu_i + \frac{1}{2}x_i(x_i + 1)\sigma_i^2 - r\right),$$

$$\sigma_i^E = \frac{V_i(t)(1 - x_i G_i V_i(t)^{-x_i-1})\sigma_i}{V_i(t) + G_i V_i(t)^{-x_i} - D_i^s}.$$

IVOL is then given by

$$IVOL_i^E(t) = \sigma_i^E\sqrt{1 - \rho_i^2} = \frac{V_i(t)(1 - x_i G_i V_i(t)^{-x_i-1})\sigma_i}{V_i(t) + G_i V_i(t)^{-x_i} - D_i^s}\sqrt{1 - \rho_i^2}. \quad (21)$$

Just as in the simple model, when all firms have identical parameters the common idiosyncratic volatility factor CIV equals a constant times the average leverage in the economy,

$$CIV(t) = \frac{1}{N}\sum_i \sigma_i^E(t)\sqrt{1 - \rho_i^2} = \sigma\sqrt{1 - \rho^2}\frac{1}{N}\sum_i \frac{V_i(t)(1 - xGV_i(t)^{-x-1})}{V_i(t) + GV_i(t)^{-x} - D_i^s}.$$

In this case, the constants x and G will be identical across firms as they only depend on the parameters. If in addition $\rho = 0$, i.e., there is no systematic risk, then the following holds.

Proposition 3. *If all firms have identical parameters and there is no systematic risk, then the* *correlation between firm-level IVOL and the CIV factor equals*

$$Cov\left(\frac{V_i(t)(1-xGV_i(t)^{-x-1})}{V_i(t)+GV_i(t)^{-x}-D_i^s}\sigma, \sigma\frac{1}{N}\sum_i \frac{V_i(t)(1-xGV_i(t)^{-x-1})}{V_i(t)+GV_i(t)^{-x}-D_i^s}\right) = \sigma^2\frac{1}{N}\left(\frac{V_i(t)(1-xGV_i(t)^{-x-1})}{V_i(t)+GV_i(t)^{-x}-D_i^s}\right)^2.$$

To see the relation between IVOL and equity beta, we define the market portfolio of equity as

$$\begin{aligned}
E_m(t) &\equiv \sum_i E_i(t) = (1-\tau_{eff})\sum_i V_i(0)(V_i(t)-D_i^s+G_iV_i(t)^{-x_i}),\\
w_i^E(t) &\equiv \frac{E_i(t)}{E_m(t)} = \frac{V_i(0)(V_i(t)-D_i^s+G_iV_i(t)^{-x_i})}{\sum_i V_i(0)(V_i(t)-D_i^s+G_iV_i(t)^{-x_i})},\\
\Delta_m(t) &\equiv (1-\tau_{eff})\sum_i(\delta_i(t)-D_i^s r).
\end{aligned}$$

The market return evolves according to

$$\frac{dE_m+\Delta_m dt}{E_m} = r_m^E dt + \sigma_m^E dW(t),$$

where

$$\begin{aligned}
r_m^E &= \sum_i \omega_i^E r_i^E,\\
&\equiv r + \frac{V_m}{E_m}(r_m^A - r) + \frac{V_m}{E_m}(-H_{m1}+H_{m2}-rH_{m3}),\\
\sigma_m^E &= \sum_i \omega_i^E \sigma_i^E \rho_i,\\
&\equiv \frac{V_m}{E_m}(\sigma_m^A - \sigma_{m4}),
\end{aligned}$$

where

$$\begin{aligned}
H_{m1} &\equiv \sum_i \omega_i^A G_i V_i^{-x_i-1} x_i\mu_i,\\
H_{m2} &\equiv \sum_i \omega_i^A G_i V_i^{-x_i-1}\left[\frac{1}{2}x_i(x_i+1)\sigma_i^2\right],\\
H_{m3} &\equiv \sum_i \omega_i^A G_i V_i^{-x_i-1},\\
\sigma_{m4} &\equiv \sum_i \omega_i^A x_i G_i V_i(t)^{-x_i-1}\rho_i\sigma_i.
\end{aligned}$$

As we can see, the nonlinear term affects both the market expected returns as well as the market volatility. The covariance between firm-level equity returns and market equity returns is given by

$$\text{Cov}\left(\frac{dE_i(t)+\Delta_i(t)dt}{E_i(t)}, \frac{dE_m(t)+\delta_m(t)dt}{E_m(t)}\right) = \frac{V_i(1-x_iG_iV_i^{-x_i-1})}{V_i+G_iV_i^{-x_i}-D_i^s}\sigma_i\rho_i\sigma_m^E.$$

And

$$\begin{aligned}\beta_i^E &= \frac{\text{Cov}\left(\frac{dE_i(t)+\Delta_i(t)dt}{E_i(t)}, \frac{dE_m(t)+\Delta_m(t)dt}{E_m(t)}\right)}{(\sigma_m^E)^2} \\ &= \frac{V_i(1-x_iG_iV_i^{-x_i-1})}{V_i+G_iV_i^{-x_i}-D_i^s}\frac{\sigma_m^A}{\sigma_m^E}\beta_i^A.\end{aligned}$$

Hence, the equity beta of firm i is again a product of the asset beta, the average leverage in the market and the leverage of the firm in question. The relation between IVOL and the equity beta still holds.

Proposition 4. *If all the firms have identical parameters, then the cross-sectional correlation between IVOL and the equity beta is positive.*

2.4 Dynamic Capital Structure

The results from the previous section allow us to study both the common IVOL factor and the alpha-IVOL relation. However, without a second refinancing boundary where the firm issues additional debt to increase leverage after firm value has increased sufficiently, the average firm will become less and less leveraged over time, making quantitative assessment of the effects much harder to do. Introducing the second boundary V_U gets around this problem. The result is a stationary process for a firm: the return process for the firm will be identical during the intervals between the firm hits any two successive V_U before hitting V_B. Again we follow the analysis in Goldstein et al. (2001).

Let us start at time $t = 0$. Again it is convenient to define a series of contingent claims. Since there are two boundaries now, we will have two contingent claims. Let $p_U(V)$ denote the present value of the contingent claim that pays \$1 when V hits V_U before hitting V_B, and $p_B(V)$ denote the present value of the contingent value that pays \$1 when V hits V_B before hitting V_U.

The value of the contingent claim $p_U(V)$ can be shown to equal

$$p_U(V) = -\frac{V_B^{-x}}{\Sigma} V^{-y} + \frac{V_B^{-y}}{\Sigma} V^{-x},$$

where

$$\Sigma \equiv V_B^{-y} V_U^{-x} - V_B^{-x} V_U^{-y}.$$

Similarly, the value of the contingent claim $p_B(V)$ is given by

$$p_B(V) = \frac{V_U^{-x}}{\Sigma} V^{-y} - \frac{V_U^{-y}}{\Sigma} V^{-x}.$$

Note that the values of these two claims are intensity variables. In other words, if all the V, V_B and V_U are scaled up by a constant factor γ, the values do not change. This is the property that we will use in the following.

Using the two contingent claim values, we can write the PV of other claims very easily. For example, for a claim that pays $\delta(t)$ as long as V does not hit V_U or V_B and zero when hit, the value is given by

$$V_{solv}^0 = V - p_B(V)V_B - p_U(V)V_U.$$

Here the superscript 0 refers to the period starting at $t = 0$ before hitting either V_U or V_B.

The total PV of the claims paying upon hitting one of the boundaries are given by

$$V_{def}^0 = p_B(V)V_B,$$
$$V_{res}^0 = p_U(V)V_U.$$

Note that the sum of the total claims is equal to the total value V of the firm

$$V_{solv}^0 + V_{def}^0 + V_{res}^0 = V.$$

The value of a claim that pays a constant interest C^0 before hitting either boundary and zero when any boundary is hit is given by

$$V_{int}^0 = \frac{C^0}{r}(1 - p_U(V) - p_B(V)).$$

Similar to the default boundary situation, different claim holders receive different claims when either boundary is hit. As before, let α and q denote the default and restructuring cost, respectively. Let us first consider the allocation of defaulting after the initial restructuring, which is similar to the previous model after the debt issuance. In other words, we first allocate the value $V - p_U(V)V_U$ to different claim holders

$$\begin{aligned}
d^0(V) &= (1-\tau_i)V_{int}^0(V) + (1-\alpha)(1-\tau_{eff})V_{def}^0(V),\\
e^0(V) &= (1-\tau_{eff})(V_{solv}^0(V) - V_{int}^0(V)),\\
g^0(V) &= \tau_{eff}(V_{solv}^0(V) - V_{int}^0(V)) + \tau_i V_{int}^0 + (1-\alpha)\tau_{eff}V_{def}^0(V),\\
bc^0(V) &= \alpha V_{def}^0(V).
\end{aligned}$$

Now we consider the restructuring branch. The discussion above is about the process after the initial restructuring at $V(0)$, which we denote as V_U^0. When the firm hits the next restructuring boundary V_U^1, we define the constant:

$$\gamma \equiv \frac{V_U^1}{V_U^0}.$$

Goldstein et al. (2001) show that V_B^1 also scales up by γ, and also $p_B^1(V_U^1) = p_B^0(V_U^0)$ and $p_U^1(V_U^1) = p_U^0(V_U^0)$. Since the optimal C^* also scales up by γ, the above split among different claims are identical in the next interval.

To summarize, initially the firm starts with $V(0) = V_U^0$. The firm then decides the capital structure choice C^0, and passes on the net proceeds of the debt issuance to to the initial equity holder. Then the firm value process follows (1) until either (1) it hits the default boundary V_B^0, or (2) it hits the restructuring boundary $V_U^1 = \gamma V_U^0$, which starts a new period.

Denote by $e(V_0)$ the present value of all claims $e^0, e^1, e^2, \ldots$:

$$\begin{aligned} e(V_0) &\equiv e^0(V_0)(1 + \gamma p_U(V_0) + [\gamma^2 p_U(V_0)]^2 + \ldots \\ &= \frac{e^0(V_0)}{1 - \gamma p_U(V_0)}. \end{aligned}$$

During a restructuring, the current debt is called back and a larger amount of new debt is issued. We assume that the debt is issued and called at par. Then the current value of the debt is equal to the PV of the cash flow before hitting V_U, $d^0(V_0)$ plus the PV of the call value, which is par

$$D^0(V_0) = d^0(V_0) + p_U(V_0) D^0(V_0).$$

It follows that

$$D^0(V_0) = \frac{d^0(V_0)}{1 - p_U(V_0)}.$$

This debt issuance will be distributed to the equity holder, adjusting for the restructuring cost q. So the present value of all future adjustment costs is given by

$$\begin{aligned} RC(V_0-) &= qD^0(V_0)(1 + \gamma p_U(V_0) + [\gamma p_U(V_0)]^2 + \ldots \\ &= \frac{qD^0(V_0)}{1 - \gamma p_U(V_0)}. \end{aligned}$$

Putting everything together, the total value of the equity is given by

$$E(V_{0-}) = \frac{e^0(V_0) + d^0(V_0) - qD^0(V_0)}{1 - \gamma p_U(V_0)}.$$

This is the sum of the present value of all future equity and debt claims net of the adjustment cost.

After the issuance of the debt, the equity value at any time during the first period is given by

$$E(V) = e^0(V) + \gamma p_U(V) E(V_{0-}) - p_U D^0(V_0). \tag{22}$$

The last term is the debt value that the firm needs to pay to the current debt holder to call back the debt.

Therefore, the following no-arbitrage condition holds, stating that the after issuance equity value equals the before issuance equity value minus the (after restructuring cost) debt issuance

$$E(V_{0+}) = E(V_{0-}) - (1-q)D^0(V_0).$$

Also around the time of the second restructuring, the following condition should hold

$$E(V_{U-}) = \gamma E(V_{0-}) - D^0(V_0).$$

The final step is to find the optimal capital structure. Namely we need to find three values, C^*, V_U, and V_B, or, equivalently, C^*, $\gamma = V_U/V_0$, $\psi = V_B/V_0$. First we find the the optimal V_B or ψ as a function of C and V_U (or γ), and then solve for C and γ. We illustrate the idea of upward-refinancing and the typical path of a firm's asset value in Figure 1.

[Insert Figure 1 here]

3 Simulations

3.1 Simulation parameter choice

We conduct simulations based on the Goldstein et al. (2001) capital structure model. We choose the same values for the main parameters as in the base case of Goldstein et al. (2001).[2]

The initial payout ratio is $\delta/V_0 = 0.035 + 0.65C/V_0$, so the drift of the payout flow rate μ equals $r - \delta/V_0 = 0.01 - 0.65(C/V_0)$.[3] In the simulation, we start with 5000 identical firms. Each

[2] We set the bankruptcy cost α at 5% as in Goldstein et al. (2001). We note that the bankruptcy cost used in Leland (1994) is 50%. Given the other parameter values, such a high bankruptcy cost yields $C^* = 0$ for all firms, suggesting that the bankruptcy cost is so high that the tax shield benefit is insufficiently large to make up for it.

[3] At any given point in time in our simulations, prices are computed under the risk-neutral measure in which any traded asset has an expected return equal to the risk-free rate. The dynamics of the firm asset value process are generated from the objective distribution, imposing a risk premium on the process of the common asset return factor W. This instantaneous drift of $\mathrm{d}V/V$ equals $r_f - (\delta/V_0) + \theta\rho\sigma$ where θ is the Sharpe ratio of the common shock $\mathrm{d}W$. We assume $\theta = 0.2$ in our simulations.

simulation run lasts for a time horizon of 50 years. We consider one time step as one day, one month is 21 days, and one year is 252 days. The initial asset value is \$100. We assume $\rho = 0.5$ for all firms. This value of ρ implies that the proportion of the total asset variance contributed by the common shock Z equals 25%. The details of the optimization and simulation algorithm are provided in Appendix D. Table 1 provides an overview of the parameter values.

3.2 Summary statistics of the simulations

To reduce the influence of a single history on our overall conclusions, we run repeated simulations with the same initial setups 100 times and report the distributions of outcomes. We report the summary statistics of the simulation results in Table 2.

[Insert Table 2]

The initial values in the first column reproduce the numbers reported in Table 3 of Goldstein et al. (2001). For example, γ is 1.7, suggesting that the firm will wait until its value rises to 1.7 times its initial value and only then is it optimal to increase the firm's leverage. The initial leverage is 0.37 which is identical for all firms at the beginning of the simulation.

The across-runs mean of the average asset value is \$347.13, and the 5^{th} and the 95^{th} percentile values are \$139.83 and \$871.46, respectively. The coupon payment grows from an initial value of \$1.85 to \$7.53 on average, given the refinancing scaling factor γ is 1.7. The average monthly return on equity is 1.06%. Whenever a firm goes bankrupt ($V_t \leq V_B$), we introduce a new firm with the same initial values into the sample in the next month, so the sample size is very stable.[4]

In Figure 2, we plot the time series of the main variables in the repeated simulations. The cross-sectional average debt value increases gradually because of the upward refinancing strategy. The equity value also increases over time. The initial leverage is 37%, which is identical for all firms, and leverage grows in approximately the first 10 years of simulation. After that, the distribution of leverage becomes stable at around 48%. In this paper, we report our main results using the samples with full simulation period. In unreported results, we repeat our tests using

[4] The average number of stocks is slightly smaller than the starting number of 5000. This gap is due to the sample filters imposed during the estimation of IVOL.

the sample which excludes the first 10 years and retains the remaining 40 years' data, and the main conclusions reached in our paper are unchanged.

[Insert Figure 2 here]

4 IVOL and CIV in the simulations

In this section, we analyze firm-level IVOL and the common factor in IVOL, CIV.

4.1 Estimating IVOL

We follow the approach in Herskovic et al. (2016) to estimate IVOL as the annualized variance of daily residual returns from an asset pricing model.[5] The asset pricing models we examine to determine residual returns are the CAPM and a five-factor principal component model (PCA), where we re-estimate the principal components each calendar year.[6] In both models, we exclude observations with equity value below $1 or with a daily equity return greater than 300% to avoid extreme estimates of IVOL. We then construct firm-year estimates of IVOL over 50 years. We exclude stocks with fewer than 100 trading days in a year. We also winsorize the top 0.5% of IVOL estimates in each year. When computing IVOL on a monthly horizon, we require a minimum of 12 trading days in the month for a stock to be included.

[Insert Table 3 here]

Consistent with Herskovic et al. (2016), the estimates from both models are very close. In Panel A of Table 3, we show that the full panel correlations between the annual IVOL estimated from the CAPM and PCA are highly correlated (99% on average across the simulation runs), and the main summary statistics such as means and standard deviations are also very close. In Panel B, we use monthly IVOL and find that the full panel correlations between the monthly IVOL estimated from CAPM and PCA are still highly correlated (96% on average). In the

[5] Many existing studies (e.g., Ang et al., 2006) compute the IVOL as the standard deviation of the residual returns. We follow the approach in Herskovic et al. (2016) and use the variance. We find either measure of IVOL does not change the results in our paper.

[6] We include only the stocks with full observations (252 days) within a year to avoid an unbalanced panel issue in the principal component analysis.

following sections of the paper, we use the CAPM-estimated IVOL in the main analysis and the PCA-estimated IVOL in robustness tests.

4.2 Common IVOL factor structure

Herskovic et al. (2016) find a strong co-movement pattern in equity IVOL in the empirical data, suggesting a common factor structure in IVOL. In this section, we find that the common factor structure in IVOL also exists in our simulation results. Within the dynamic capital structure model, the only possible channel to generate co-movement is financial leverage. We follow Herskovic et al. (2016) and Duarte et al. (2014) and conduct three analyses to examine the common factor structure in IVOL and its relation with financial leverage.

4.2.1 Average IVOL in the time-series and pairwise correlations

In our first analysis, we visually show the strong co-movement in IVOL by plotting the time-series of average IVOL of stock portfolios. In each simulation run, we sort the stocks into quintiles by different firm characteristics, such as market capitalization and financial leverage; then we compute the equally-weighted average IVOL for each quintile. In Figure 3, we plot the time-series of average IVOL for the portfolios sorted by size (Figure 3a) and by leverage (Figure 3b) in one simulation run. The average IVOL of all size quintiles shows a clear co-movement pattern. The average IVOL of leverage-sorted portfolios shows a very similar co-movement pattern, except for the lowest leverage quintile whose average IVOL appears to be flat over the full sample period.

[Insert Figure 3 here]

Next, we investigate the IVOL co-movement in a more formal test by examining the average pairwise correlations between the average IVOL of the portfolios sorted by size or leverage. In Table 4, we report the results based on repeated simulation runs. We follow the procedure used to construct Figure 3. In each simulation run, we sort the firms by size (or by leverage) into quintiles and compute the average IVOL for each quintile in each year, then we compute the pairwise correlations of the average IVOL between the quintiles. We repeat this procedure in repeated simulation runs. Next, we compute the mean, 5^{th} and 95^{th} percentile values of the correlations for each quintile pair over the repeated simulation runs. For example, the number in column 4 and row 1 in Panel A1 (size) of Table 4 shows that the mean correlation of the average IVOL between the largest and second-largest size quintile is 0.93; the two numbers in brackets report that the 5^{th} and 95^{th} percentile values of the correlation are 0.84 and 0.98.

[Insert Table 4 here]

In Panel A1 (size), the average pairwise correlations among average IVOL of the five size quintiles range from 0.81 to 0.96, suggesting that the co-movement in IVOL is very strong. In Panel A2 (leverage), the average pairwise correlations are also very high among the leverage quintiles 2 to 5. However, the pairwise correlation between the lowest leverage quintile with the other quintiles is close to zero on average and is insignificantly different from zero. This finding suggests that for the lowest leverage quintile, the average IVOL is essentially constant and remains unchanged in the time series. It also suggests that the leverage in the lowest leverage quintiles does not change over time. This result supports our hypothesis that time-varying financial leverage drives the co-movement in IVOL. In Panel B, we repeat the same analysis using monthly IVOL and the conclusion is unchanged.

In Equation (21), we show how IVOL is determined by the firm's financial leverage at the beginning of the period, assuming a constant asset return volatility. Similarly, the equally-weighted average IVOL of a portfolio formed in this section reflects the average leverage of stocks in a portfolio. Therefore the co-movement in average IVOL is a result of market-wide time-varying leverage.

4.2.2 Common IVOL factor in explaining individual IVOL

In our second test, we follow Herskovic et al. (2016), and run firm-by-firm time-series regressions regressing a firm's IVOL on the common factor in IVOL (CIV). CIV is measured as the equally-weighted IVOL in each cross-section. Hence CIV directly reflects the market equally-weighted average leverage in our setup.

In each simulation run, we run the time-series regression on each firm and compute the R_2, which indicates how much of the time variation in the firm's IVOL can be explained by the single common factor CIV.[7] We average the IVOL in the cross-section in each simulation run and then compute the mean, 5$^{\text{th}}$ and 95$^{\text{th}}$ percentile values of average across the simulation runs. In Table 5 we report results for this levered sample for both annual and monthly IVOL. To examine the relation between the common factor structure in IVOL and financial leverage, we redo this analysis for an unlevered sample. In the unlevered sample, we use the same initial parameter values and the same dynamics of V, but we force C and V_B equal to zero for all stocks throughout the simulation horizon, so the equity value E of a firm equals its asset value V.[8]

[Insert Table 5 here]

In Table 5, Panel A row *Levered*, we show that when firms are levered, the common factor in IVOL explains 25% of the time variation in individual IVOL on average, and the 5$^{\text{th}}$ and 95$^{\text{th}}$ percentile values of average R^2 in repeated simulations are 18% and 31% respectively.[9] This result suggests that a substantial proportion of the variations in individual IVOL can be explained by a single common factor. In Figure 4, we plot the distribution of average R^2 in the repeated simulations and confirm that a common factor structure of IVOL exists in our levered sample.

[Insert Figure 4 here]

As shown in Table 5, when we use the unlevered sample the average R^2 reduces substantially to only 2%, suggesting that the time variation in individual IVOL is unable to be explained by a single factor, and also suggesting there is no common factor structure in IVOL when the firms are unlevered. In Panel B, we confirm our finding using monthly IVOL (not examined in Herskovic et al., 2016). The magnitude of average R^2 for the levered sample is smaller (13%) than the value in annual data, while the average R^2 for the unlevered sample drops to zero.

[7] Note that in our illustration model, e.g., Equation (21), IVOL is the product of leverage and asset return volatility. If the asset return volatility is assumed to be constant and identical across firms, then CIV is the equally-weighted average leverage across all firms multiplied by the constant asset return volatility.

[8] In the unlevered sample, we force the debt equal to zero to study the effect of financial leverage in determining the common factor structure in IVOL. The firm does not achieve an optimal capital structure in this setup.

[9] The average R^2 from our simulation data is smaller than the figure of 35% reported in Herskovic et al. (2016) using empirical data. The magnitude of R^2 depends on the initial parameter values used in our simulation. In general, higher leverage would lead to a higher average R^2. In this paper it is not our focus to match the empirical results.

4.2.3 Determinants of the common IVOL factor

In the third test, we examine the determinants of the common factor in IVOL directly. Following the methodology in Duarte et al. (2014), we run a time-series regression at the market level. The left-hand-side variable is the innovation in the common factor in IVOL (ΔCIV_t), the right-hand-side variables are the innovations in market average financial leverage ($\Delta Lev_{m,t}$), the innovations in market value-weighted equity return variance ($\Delta\sigma^2_{m,t}$) and the innovations in market average credit spread ($\Delta CS_{m,t}$). $\Delta\sigma^2_{m,t}$ and $\Delta CS_{m,t}$ are two variables identified in Duarte et al. (2014) that explain the innovations in CIV. The innovations are measured as the monthly or annual changes in each variable, depending on the frequency of data we use in the analysis.

$$\Delta CIV_t = \alpha + \beta_1 \Delta\sigma^2_{m,t} + \beta_2 \Delta Lev_{m,t} + \beta_3 \Delta CS_{m,t} + \epsilon_t. \tag{23}$$

In the Goldstein et al. (2001) model, the credit spread is a direct translation of financial leverage. In our simulation data, the correlations between the market average leverage and the market average credit spread are more than 99% both in levels and in first differences (see Table 12). To avoid the issue of multicollinearity, we run regressions with leverage and credit spread separately. In Table 6, we report the mean, 5$^{\text{th}}$ and 95$^{\text{th}}$ percentile values of the estimated regression coefficients. In addition, we report the percentage of simulation runs in which the estimated coefficient is significantly different from zero at the 5% level of significance for each variable.

[Insert Table 6 here]

Panel A of Table 6 shows the result with annual data. Consistent with the finding in Duarte et al. (2014), we find that the innovations in market equity return variance and credit spread are positively related to ΔCIV. The innovations in financial leverage, which are our main focus, also show a clear positive relation with ΔCIV. The mean of the coefficient of $\Delta Lev_{m,t}$ is 0.43, while the 5$^{\text{th}}$ percentile value is 0.19. In Panel B, we use monthly data and find an even stronger effect of financial leverage and credit spread in determining the CIV. For example, in 100% of all simulation runs, the estimated coefficient on $\Delta Lev_{m,t}$ is significant at 5%, and the magnitude of the coefficient is very similar to Panel A. Conversely, the magnitude of the coefficient on market return variance is much smaller. This result suggests that time-varying financial leverage better captures the changes in CIV when we use higher-frequency data. This result is to be expected as over longer horizons the refinancing boundary limits the cross-sectional variability in the leverage ratios.

In summary, we conduct three analyses which show that the common factor structure in IVOL exists in a standard capital structure model and the common factor structure in IVOL is driven by financial leverage. Time-varying financial leverage determines the measure of IVOL and drives the comovement in IVOL in the cross-section. The common factor structure in IVOL totally disappears if all firms are purely equity-financed (zero leverage).

4.3 Common IVOL factor, lagged IVOL and expected stock returns

We also explore two IVOL-related cross-sectional pricing effects. Herskovic et al. (2016) find that the exposure to CIV shocks is negatively priced in the cross-section of stocks. Ang et al. (2006) find that the stocks with high lagged IVOL earn puzzlingly low future returns — this finding is documented as the IVOL puzzle. Herskovic et al. (2016) find that both anomalies co-exist. We examine jointly the pricing effect of exposure to CIV shocks and lagged IVOL in our numerical analysis. We start by exploring the average returns on the portfolios sorted by firms' exposure to CIV shocks or lagged IVOL. We then conduct formal firm-level Fama-MacBeth regressions in which exposure to CIV shocks and lagged IVOL are the key independent variables. Following Herskovic et al. (2016) and Ang et al. (2006), we conduct our analysis in this section using monthly IVOL. As before, IVOL is estimated as the idiosyncratic variance of residual returns from CAPM regressions using daily returns within a given month, while CIV is measured as the equally-weighted average of IVOL in the cross-section within a given month.

4.3.1 CIV as a pricing factor

We follow Herskovic et al. (2016) to estimate the exposure to CIV shocks. The shocks to CIV are measured as the monthly changes.

$$r_{i,t} - r_{f,t} = \alpha_{i,t} + \beta_{CIV,i} \Delta CIV_t + \epsilon_{i,t}. \tag{24}$$

where t denotes a month. $r_{f,t}$ is the monthly risk-free rate. We estimate the regression (24) on a 60-month rolling window.[10]

We then sort stocks into quintiles based on their exposures to CIV shocks (β_{CIV}) each month and form a portfolio of the stocks in each quintile and hold that portfolio in the following month. Table 7 reports the average raw returns on each portfolio, as well as the return on a strategy that goes long on the highest β_{CIV} quintile and short on the lowest β_{CIV} quintile. Consistent with the finding in Herskovic et al. (2016), the stocks with highest β_{CIV} earn a significantly lower average return in the holding month than the stocks with lowest β_{CIV}. On average, the equally-weighted and value-weighted returns of the long-short portfolio are -0.84% and -0.25% respectively, across repeated simulation runs.

[Insert Table 7 here]

We also find that the portfolio with highest β_{CIV} have lower CAPM beta, lower IVOL, higher equity value and lower leverage than the portfolio with lowest β_{CIV}. There is a negative correlation between β_{CIV} and IVOL, suggesting that higher-IVOL stocks have lower β_{CIV} on average. This negative correlation is consistent with the observation in Herskovic et al. (2016) that the correlation between the two is -6%.[11]

We relate the negative pricing effect of β_{CIV} to the cross-sectional difference in firms' financial leverage. As we discussed in our illustrative model, if the economy does poorly (well), then all firms will tend to become more (less) levered, but this effect is not uniform. Following this idea, we use the table below to illustrate our explanation.

A negative shock in the market asset portfolio causes a positive shock in market leverage and hence a positive shock in CIV. The high-leverage firms suffer large losses in equity value when the market does poorly; hence, when we regress the equity returns on ΔCIV_t, the estimate of β_{CIV} is negative and large in magnitude. An example is the lowest β_{CIV} quintile in Table 7. In turn, the leverage of such firms will further increase because of the large loss in equity value in the current period *t*. High leverage implies a high expected return in the next period *t+1*. The upshot is a negative relation between β_{CIV} and expected stock returns. In Section 4.3.3,

[10] We require a minimum of 36 months of observations for a stock to be included in any given 60-month period.
[11] The correlation in Herskovic et al. (2016) is likely to be a noisier measure of the true correlation than ours, so we expect its magnitude to be smaller.

Financial leverage, exposure to CIV shocks, and expected stock returns

This table illustrates how financial leverage is related to the negative relation between the exposure to CIV shocks and expected stock returns. $r^A_{m,t}$ is the market asset return in period t. CIV is the common IVOL factor measured as the equally-weighted average of IVOL in the cross-section. ΔCIV is the shock to CIV. $r^E_{i,t}$ is the individual equity return in period t. $E(r_{t+1})$ is the expected equity return. $\beta_{CIV,i,t}$ is the exposure to CIV shocks.

	t	$t+1$
Market	negative $r^A_{m,t}$	
	$Leverage_t \uparrow$	
	$CIV_t \uparrow (+\Delta CIV_t)$	
Higher-levered firms	more negative $r^E_{i,t}$	high $E(r_{t+1})$
	more negative $\beta_{CIV,i,t}$	
Lower-levered firms	less negative $r^E_{i,t}$	low $E(r_{t+1})$
	less negative $\beta_{CIV,i,t}$	

we report the results of Fama-MacBeth regressions to examine further the relation between financial leverage and the negative pricing effect of the exposure to CIV shocks.

4.3.2 The IVOL puzzle effect

In this section we examine the cross-sectional relation between lagged IVOL and equity returns. Each month, we sort the stocks into quintiles by IVOL and hold the portfolios in the following month. In Table 8, we report the average raw returns on each portfolio, as well as the return on a strategy that goes long on the highest-IVOL quintile and short on the lowest-IVOL quintile. Ang et al. (2006) find that the stocks with high IVOL receive puzzlingly low future returns. Inconsistent with Ang et al.'s result, we find a positive relation between IVOL and cross-sectional equity returns in our numerical analysis based on the Goldstein et al. (2001) capital structure model. This is to be expected since both expected return and IVOL are closely and positively related to a firm's leverage, as also argued in Bhandari (1988). As reported in Table 8, the leverage of the high-IVOL quintile is 0.68 compared to 0.36 for the low-IVOL quintile. The equally-weighted and value-weighted returns of the long-short portfolio are 2.98% and 1.27%, respectively, averaged over repeated simulation runs. All intervals between the 5^{th} and 95^{th} percentile values of the long-short portfolio returns (both equally-weighted and value-weighted) consist of positive values; hence they do not include zero. We also find that the portfolios with highest IVOL have more negative β_{CIV}, higher CAPM beta, lower equity value, higher leverage and a higher credit spread than the portfolios with lowest IVOL.

[Insert Table 8 here]

The positive relation between IVOL and cross-sectional returns is persistent and is not driven by short-term return reversal effects. Fu (2009) and Huang, Liu, Rhee, and Zhang (2010) relate the observed cross-sectional pricing effect of IVOL in the empirical data to short-term return reversals. By examining the longer-term performance of the IVOL-sorted portfolios, we show that the positive IVOL-return relation in our result is not driven by short-term return reversals. In Table 9, we show that the portfolio with highest IVOL earns significantly higher average returns in the portfolio formation month, the holding month, as well as in a longer holding horizon up to at least a quarter.[12] All the long-short portfolio monthly returns are positive, ranging from 2.81% to 2.98% during the holding months $t + 1$ to $t + 3$. The intervals between the 5^{th} and 95^{th} percentile values of the long-short portfolio returns over repeated simulations consist of values that are substantially larger than zero.

[Insert Table 9 here]

As discussed in Section 4.4.3.1, the highly levered firms tend to have the most negative β_{CIV}. Given that IVOL is positively related to leverage, which is negatively related to β_{CIV}, it follows that there should be a negative correlation between IVOL and β_{CIV}. In addition, the negative correlation between the two also reconciles the fact that both high IVOL and low β_{CIV} predict a high expected return because of high leverage. On the other hand, this negative relation between IVOL and β_{CIV} does not help to explain the IVOL puzzle in the empirical data. Hence our analysis agrees with the conclusion in Herskovic et al. (2016) that the negative pricing effect of both and IVOL co-exist.

4.3.3 Fama-MacBeth regressions

In the previous two subsections, we showed that the exposure to common IVOL factor shocks (β_{CIV}) is negatively priced, and lagged IVOL is positively priced in the cross-section within

[12] In Appendix B and Table 13, we also test the persistence of the IVOL rankings. We focus on the high-IVOL stocks, and report the subsample returns in the future depending on whether the stocks persistently stay in the top IVOL quintile or not. We find that around 50% of the high-IVOL stocks persistently stay in the top IVOL quintile for at least 3 months after portfolio formation. The fact that these stocks earn the highest returns in each of the holding months suggests that the positive relation between future returns and IVOL is highly persistent.

the model. We also discussed the relation between financial leverage and the pricing effects of β_{CIV} and IVOL. In this section, we conduct more formal tests. In Table 10, we report the correlations between the main variables. We find that expected equity returns are negatively related to β_{CIV} and size, but positively related to IVOL, leverage, credit spread and β_{CAPM}. The full panel correlation between β_{CIV} and IVOL is -0.18 on average in repeated simulation runs. The average correlation between leverage and β_{CIV} is -0.21. In our illustrative model, we showed that financial leverage determines both the measure of IVOL and β_{CAPM}. Consistent with this result, we find that financial leverage is strongly positively related to the measure of IVOL and β_{CAPM}; the correlations are 0.73 and 0.56, respectively. Empirical studies such as Bartram, Brown, and Stulz (2016) find no significant correlations between financial leverage and IVOL. The difference between the empirical finding and the theoretical implication, as Choi and Richardson (2016) point out, is due to failing to control for asset volatility. Doshi et al. (2016) also note that the empirical studies use levered equity returns whereas theoretical implications are valid for unlevered returns.

[Insert Table 10 here]

Next we conduct Fama-MacBeth type cross-sectional regressions to examine formally the ability of β_{CIV} and IVOL to explain the cross-sectional stock returns. Table 11 shows the risk premia estimates from the Fama-MacBeth regressions with different independent variables. The first column reports the univariate test result with β_{CIV}. The estimated coefficient of β_{CIV} is -0.32 on average, with all values between the 5^{th} and 95^{th} percentile values below zero. In 99% of the repeated simulation runs, the coefficient is significant at the 5% level. In Column (2), we include IVOL which is significantly priced in the cross-section with a positive sign. The magnitude of the β_{CIV} coefficient is reduced, and its sign even turns positive. In Column (3), we find that financial leverage is significantly priced with a positive sign, and financial leverage largely captures the negative pricing effect of β_{CIV}. The average of the β_{CIV} coefficient is reduced to -0.01, with zero being included in the interval between the 5^{th} and 95^{th} percentile values. In only 14% of repeated simulation runs is the β_{CIV} coefficient significantly different from zero at the 5% level.

[Insert Table 11 here]

The negative pricing effect of β_{CIV} in the cross-section can be explained by financial leverage. This finding supports our analysis that low β_{CIV} stocks tend to be highly levered. On the other hand, IVOL is positively priced, because by construction in the Goldstein et al. (2001) capital structure model, IVOL is directly determined by financial leverage. In this context, it is not surprising to find that IVOL also captures the negative pricing effect of β_{CIV} through the link with financial leverage.

5 Conclusions

We provide an explanation of the common factor structure in IVOL through time-varying financial leverage. The only requirement is that firms cannot completely and immediately adjust their leverage to remove the natural correlation between leverage and the return on the economy. In a simple model that assumes a single factor structure in firms' asset returns, we show that IVOL reflects the time variation in leverage, even when the residual equity returns are uncorrelated and there is no common factor structure in the fundamental cash flow idiosyncratic volatility. When the economy performs poorly (well), all firms tend to become more (less) levered and firm-level IVOL will tend to increase (decrease) together.

We quantitatively examine the relation between financial leverage and the common factor in IVOL using a variant of the Goldstein et al. (2001) capital structure model. Under reasonable initial parameter settings, we show that a single common factor in IVOL explains a substantial portion of the time variation in firm-level IVOL. Additionally, we find that for portfolios sorted by financial leverage the average IVOL are strongly correlated with each other except for the portfolio with lowest leverage. In the market-level time-series regression, we find that the innovation in market average financial leverage significantly determines the innovations in the cross-sectional average IVOL (CIV). Importantly, when we use a sample of unlevered firms in the same set up, IVOL does not display a common factor structure at all.

We also show in our numerical analysis that the exposure to CIV shocks is negatively priced in the cross-section but IVOL is positively priced. Financial leverage explains these two pricing effects jointly. IVOL is a positive monotonic transformation of the firm's leverage; while the firm's exposure to CIV shocks has a negative relation with leverage. Hence firms with high leverage tend to have high IVOL but low exposure to CIV shocks and have high expected returns, and vice versa.

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Appendix A Proofs

A.1 Proofs of results in Section 2.2

The detailed proofs can be found in Goldstein et al. (2001). Here we mainly sketch the process of the derivation.

For any claim on the asset value with intermediate payoff rate C, the value of the claim, $F(V,t)$ satisfies the following PDE

$$\frac{1}{2}\sigma^2 V^2 F_{VV} + \nu V F_V - rF + F_t + C = 0. \tag{25}$$

Consider that the firm only issues a consol debt, thus the value function is time-invariant. The resulting ODE becomes then

$$\frac{1}{2}\sigma^2 V^2 F_{VV} + \nu V F_V - rF + C = 0. \tag{26}$$

For a homogeneous ODE with the term C, the solutions are of the form $F(V) = A_1 V^{-y} + A_2 V^{-x}$, where

$$x = \frac{1}{\sigma^2}\left[\left(\nu - \frac{\sigma^2}{2}\right) + \sqrt{\left(\nu - \frac{\sigma^2}{2}\right)^2 + 2r\sigma^2}\right] > 0,$$
$$y = \frac{1}{\sigma^2}\left[\left(\nu - \frac{\sigma^2}{2}\right) - \sqrt{\left(\nu - \frac{\sigma^2}{2}\right)^2 + 2r\sigma^2}\right] < 0.$$

The only boundary in this set up is the default boundary, V_B. For convenience, define the price of a claim that pays \$1 when the firm defaults as p_B. Since there is no intermediate payment for this claim, the general solution, as above, is given by

$$p_B(V) = A_1 V^{-y} + A_2 V^{-x}. \tag{27}$$

Consider the boundary conditions, $\lim_{V\to\infty} p_B(V) = 0, \lim_{V\to V_B} p_B(V) = 1$, the price is then given by

$$p_B(V) = \left(\frac{V}{V_B}\right)^{-x}. \tag{28}$$

Given this result, we now study the values of equity, debt and government claims, respectively. We start by considering the holder receiving all the payouts as long as the firm does not default, and zero in case of default. The value of the claim for this holder, denoted as V_{solv}, should be equal to the difference between the total value V and the value in case of default,

$$V_{solv} = V - V_B p_B(V). \tag{29}$$

Next we consider the holder receiving all the coupons, constant C, if the firm does not default, and zero if so. The value of a console bond with constant coupon C is given by C/r (recall we are in the risk-neutral world). So the value of the holder, denoted as V_{int}, is given by:

$$V_{int} = \frac{C}{r}[1 - p_B(V)]. \tag{30}$$

As such, we can easily write out the values accruing to the different claim holders who receive payments when there is no default, and zero otherwise

$$\begin{aligned}
E_{solv}(V) &= (1 - \tau_{eff})(V_{solv} - V_{int}), \\
G_{solv}(V) &= \tau_{eff}(V_{solv} - V_{int}) + \tau_i V_{int}, \\
D_{solv} &= (1 - \tau_i)V_{int},
\end{aligned}$$

where

$$\tau_{eff} = (1 - \tau_c)(1 - \tau_d), \tag{31}$$

and τ_i, τ_d, τ_c are the tax rates for interest, dividends and corporate profits, respectively.

We next study the value of different claim holders when there is a default. Recall that the PV of contingent claim paying \$1 in case of default is given by $p_B(V)$. Given that the firm value equals V_B at default, the total PV of the claim to default is given by

$$V_{def}(V) = V_B p_B(V). \tag{32}$$

The default value V_B will be distributed to three parties: debt holders, government and bankruptcy cost. Denote the proportional bankruptcy cost by $\alpha > 0$, so that the PV of the bankruptcy cost, $BC_{def}(V)$, equals

$$BC_{def}(V) = \alpha V_{def}(V). \tag{33}$$

The remaining value, $(1-\alpha)V_B$, is distributed between the debt holder and the government through tax. Here we assume that the tax rate is charged as a dividend payment. Then the PVs of the default claims for the debt holder and government, respectively, equal

$$D_{def}(V) = (1-\alpha)(1-\tau_{eff})V_{def}(V),$$
$$G_{def}(V) = (1-\alpha)\tau_{eff}V_{def}(V).$$

The equity holder, of course, receives nothing in bankruptcy. So the total value of equity after debt issuance is given by

$$E(C,V;V_B) = E_{solv} = (1-\tau_{eff})\left[V - V_B\left(\frac{V}{V_B}\right)^{-x} - \frac{C}{r}\left(1-\left(\frac{V}{V_B}\right)^{-x}\right)\right].$$

To obtain the optimal default value V_B, we impose smooth-pasting condition as usual,

$$0 = \left.\frac{\partial E}{\partial V}\right|_{V=V_B}.$$

Then the optimal V_B^* is given by

$$V_B^* = \frac{x}{1+x}\frac{C}{r} \equiv \lambda\frac{C}{r}, \text{ with } \lambda = \frac{x}{1+x}. \tag{34}$$

Note that the optimal default boundary is a function of the coupon payment C.

Given this, we can rewrite the value of the equity as

$$E(V,C,V_B(C)) = (1-\tau_{eff})\left[V + \frac{1}{1+x}\lambda^x\left(\frac{C}{r}\right)^{x+1}V^{-x} - \frac{C}{r}\right].$$

The debt holder receives C as long as the firm does not default, and $(1-\alpha)(1-\tau_{eff})V_B$ upon default. So the value of the debt at any time is given by

$$D = D_{solv} + D_{def},$$

Let's consider the optimal coupon. Before the issuance of the debt, the equity holder, owning the whole firm, decides to issue the debt at the market value $D(V_0, C, V_B(C))$. There is also a restructuring cost q, so the net value to the equity holder at time $t = 0$ is given by

$$(1-q)D(V_0, C, V_B(C)) + E(V_0, C, V_B(C)). \tag{35}$$

The equity holder chooses the optimal capital structure (i.e., coupon C) by maximizing the above value. It follows that:

$$C^* = \frac{rV_0}{\lambda}\left[\left(\frac{1}{1+x}\right)\left(\frac{A}{A+B}\right)\right]^{\frac{1}{x}}, \tag{36}$$

where

$$\begin{aligned} A &= (1-q)(1-\tau_i) - (1-\tau_{eff}), \\ B &= \lambda(1-\tau_{eff})(1-(1-q)(1-\alpha)). \end{aligned}$$

Appendix B Correlations and IVOL persistency

In the capital structure model in Goldstein et al. (2001), the credit spread is a direct positive transformation of financial leverage. In Table 6, we examine the determinants of CIV at the market level using the market average credit spread and the market average financial leverage as dependent variables. In Table 12, we examine the correlations between the market average credit spread and the market average financial leverage both in levels and in differences. We do not include the two variables in our regressions simultaneously because they are very highly correlated.

In our numerical analysis based on the capital structure model in Goldstein et al. (2001), we find a significant positive relation between IVOL and expected stock returns. In Table 13, we further examine the persistence of the IVOL ranking and returns for the high-IVOL stocks over longer holding horizons. We show that more than 50% (507.9 out of 985.4) of the stocks sorted into the top IVOL quintile in the current month remain in the top quintile in each of the next three months, and that this group of stocks earns higher average returns than the other stocks which initially were also part of the top quintile.

Appendix C Symbols

Symbol	**Description**
Superscripts	
A	Assets (of a firm or the market portfolio)
E	Equity (of a firm or the market portfolio)
Subscripts	
i	A firm
m	The market portfolio
t	One period
f	Risk-free
Symbols in Section 4.2	
$^{A}_{i}$	Asset beta of firm i
$r^{A}_{m,t}$	Period t return on the market portfolio of all assets
$r^{A}_{i,t}$	Period t return on firm i's assets
$r^{E}_{i,t}$	Period t return on firm i's equity
r_f	Risk free rate
$e^{A}_{i,t}$	The idiosyncratic return of the period t return on the assets of firm i measured relative to the return on the market portfolio of all assets
$^{A}_{i}$	The idiosyncratic volatility of return on the assets of firm i
$Er^{A}_{m,t}$	Expected asset returns
$V^{A}_{i,t-1}$	The value of firm is assets at the start of period t
$D^{A}_{i,t-1}$	The value of the debt of firm i at the start of period t
λ	The weight put on the desired firm-specific asset-to-equity ratio
Symbols in Section 4.3	
$\mathrm{d}Z$	Brownian motions common factor Z under the risk-neutral probability measure
$\mathrm{d}\tilde{Z}$	Brownian motions common factor Z under the objective probability measure
$\mathrm{d}W_i$	Brownian motions firm-specific factor Z under the risk-neutral probability measure
$\mathrm{d}(\tilde{W}_i)$	Brownian motions firm-specific factor Z under the objective probability measure

Symbol	**Description**
δ_i	Asset payout flow of firm i
ρ_i	the proportion of the total asset variance contributed by the common shock Z
r_f	Risk-free rate
θ	Sharpe ratio of the exposure to the common shock dZ
σ	Asset return volatility
α	Bankruptcy cost
ϵ	Tax shield effective rate when bankruptcy
q	Restructuring (refinancing) cost rate
μ	Drift of the payout flow rate
γ	Upward refinancing scaling factor
n	The number of refinancing rounds that have taken place
τ_i	Personal interest income tax rate
τ_d	Personal dividend income tax rate
τ_c	Corporate tax rate
C	Coupon payment
C_0	Initial coupon payment
C_1	The coupon payment after the first refinancing when the firm remains solvent
C_n	The coupon payment after the nth refinancing when the firm remains solvent
$V_{i,0}$	Initial asset value of firm i
V_0	Initial asset value of a firm
V_t	Asset value at time t
V_Z	Asset value at the beginning of each period after the previous refinancing
V_B	Bankruptcy threshold
$V_{B.0}$	Bankruptcy threshold before bankruptcy or the first refinancing
$V_{B.n}$	Bankruptcy threshold after the nth refinancing and before bankruptcy or the next refinancing
V_U	Upward refinancing threshold
$V_{U,0}$	Upward refinancing threshold before bankruptcy or the first refinancing
$V_{U,n}$	Upward refinancing threshold after the nth refinancing and before bankruptcy or the next refinancing
$P_U(V)$	The present value of a claim that pays \$1 contingent on firm value reaching V_U
$P_B(V)$	The present value of a claim that pays \$1 contingent on firm value reaching V_B
$e^0(V_0)$	The period 0 present values to the dividends after the initial debt issuance, but before bankruptcy or next refinancing
$d^0(V_0)$	The period 0 present values to the coupon payments after the initial debt issuance, but before bankruptcy or next refinancing
$g^0(V_0)$	The period 0 present values to the taxes after the initial debt issuance, but before bankruptcy or next refinancing
$e^0(V_0)$	The period 0 present values to the dividends after the initial debt issuance, but before bankruptcy or next refinancing
$d^0(V_0)$	The period 0 present values to the coupon payments after the initial debt issuance, but before bankruptcy or next refinancing
$g^0(V_0)$	The period 0 present values to the taxes after the initial debt issuance, but before bankruptcy or next refinancing
$D^0(V_0)$	Initial debt value
$E(V_0)$	Initial equity value
$E(V_{0-})$	Initial equity value before issuing any debt
$D(V_t)$	Debt value at time point t
$E(V_t)$	Equity value at time point t
$E(V_{Z-})$	Equity value immediate before the next refinancing

Appendix D Setups and Simulation Algorithm

We study a cross-section of firms whose capital structure decisions follow the Goldstein et al. (2001) upward refinancing capital structure model. In this section, we follow their notation. Given the input parameters, the first step is to numerically solve for the initial optimal C^*, γ^* and V_B^* endogenously. The second step is to simulate the firm's asset value over a long horizon (daily data over 50 years) and repeat this for a cross-section of firms. If a firm defaults, a new firm is introduced with the same characteristics. The (unbalanced) panel of daily firm asset returns that results is then used to compute equity returns, IVOL and other metrics analyzed in this paper. The details of the two steps are as follows.

Step 1 Numerical solution

At time $t = 0$ (beginning), management maximizes the equity value before issuing the initial debt,

$$E(V_{0-}) = \frac{e_0(V_0) + d_0(V_0) - qD_0(V_0)}{1 - \gamma p_U(V_0)}.$$

Then, firm management chooses the initial coupon payment C, upward refinancing multiplication factor γ and bankruptcy boundary V_B such that the total equity value $E(V_0)$ for arbitrary V during any time after the initial debt issuance and before the first upward refinancing is maximized. $d_{V_0}^0$ and $e_{V_0}^0$ are the period 0 present values to the coupon payment and dividends, respectively, after the initial debt issuance, but before bankruptcy or next refinancing. The symbols are listed in Appendix C. The first-order-condition (FOC) is

$$\begin{aligned}
\frac{\partial E(V)}{\partial V}|_{V=V_B} &= 0, \\
&= [\gamma E(V_{0-}) - D_0(V_0)] \cdot \left(\frac{V_B^{-X}}{M} y V^{-y-1} - \frac{V_B^{-y}}{M} x V^{-x-1} \right) \\
&- B_1 y V^{-y-1} - B_2 x V^{-x-1} + K, \\
&= [\gamma E(V_{0-}) - D_0(V_0)] \cdot \frac{y - x}{M} V_B^{-x-y-1} - B_1 y V_B^{-y-1} - B_2 x V_B^{-x-1} + K,
\end{aligned}$$

where x and y are defined as

$$x = \frac{1}{\sigma^2}\left[(\mu - \frac{\sigma^2}{2}) + \sqrt{(\mu - \frac{\sigma^2}{2})^2 + 2r\sigma^2}\right],$$
$$y = \frac{1}{\sigma^2}\left[(\mu - \frac{\sigma^2}{2}) - \sqrt{(\mu - \frac{\sigma^2}{2})^2 + 2r\sigma^2}\right].$$

Furthermore, the upward refinancing threshold equals

$$V_U = \gamma V_0,$$

the market value of debt

$$D_0(V_0) = \frac{d_0(V_0)}{1 - p_U(V_0)}.$$

$p_B(V)$ is the present value of a claim that pays \$1 contingent on firm value reaching V_B; $p_U(V)$ is the present value of a claim that pays \$1 contingent on firm value reaching V_U (before reaching V_B),

$$p_U(V) = -\frac{V_B^{-x}}{M}V^{-y} + \frac{V_B^{-y}}{M}V^{-x},$$
$$p_B(V) = \frac{V_U^{-x}}{M}V^{-y} - \frac{V_U^{-y}}{M}V^{-x},$$
$$M = V_B^{-y}V_U^{-x} - V_U^{-y}V_B^{-x}.$$

The total value of the firm can then be written in terms of its components as

$$V_{solv,0}(V) = V - p_U(V)V_U - p_B(V)V_B,$$
$$V_{int,0}(V) = \frac{C_0}{r}[1 - p_U(V) - p_B(V)],$$
$$V_{def,0}(V) = p_B(V)V_B.$$

$V_{solv,0}(V)$ is the total present value of claims of equity holders, debt holders and government to the firm while the firm is solvent. $V_{int,0}(V)$ is the present value of the claim to interest payment while the firm is solvent. $V_{def,0}(V)$ is the present value of the default claim.

The value of debt and equity at any point between two consecutive restructuring epochs equals

$$
\begin{aligned}
d_0(V) &= (1-\tau_i)V_{int,0}(V) + (1-\alpha)(1-\tau_{eff})V_{def,0}(V) \\
e_0(V) &= \left[B_1 V^{-y} + B_2 V^{-x} + (KV - H\frac{C}{r})\right] * (V < V_*) \\
&+ \left[A_1 V^{-y} + A_2 V^{-x} + K(V - \frac{C}{r})\right] * (V > V_*)
\end{aligned}
$$

Following Goldstein et al. (2001), the value of equity loses some of the tax shield when V drops below a specified level V_*, to reflect the fact that a firm that is not profitable loses part of its tax shelter. As in Goldstein et al, we assume the threshold for the tax shield not be fully captured to be $V_* = 17C$.

We define the following helper variables.

$$
\begin{aligned}
K &= 1 - \tau_{eff}, \\
H &= 1 - \epsilon\tau_{eff}, \\
B_1 &= (HC/r - KV_B)c_4/M + K(V_U - C/r)c_2/M \\
&\quad + (K-H)(C/r)c_2(xc_3/c_5 - yc_4/c_6)/[(x-y)M], \\
B_2 &= (HC/r - KV_B)/c_2 - B_1c_1/c_2, \\
A_1 &= B_1 - x(K-H)(C/r)/[(y-x)c_5], \\
A_2 &= B_2 - y(K-H)(C/r)/[(x-y)c_6], \\
c_1 &= V_B^{-y}, c_2 = V_B^{-x}; c_3 = V_U^{-y}, c_4 = V_U^{-x}; c_5 = V_*^{-y}, c_6 = V_*^{-x}.
\end{aligned}
$$

Finally, we also impose boundary conditions on the decision variables,

$$
\begin{aligned}
&0 < V_B < V_0, \\
&V_B < V_*, \\
&\gamma > 1, \\
&C \geq 0.
\end{aligned}
$$

We numerically solve for the initial optimal C^*, γ^* and V_B^* and then we can also compute the main variables as follows.

The initial total equity value of a levered firm

$$E(V_0) = \gamma p_U(V_0)E(V_{0-}) + e_0(V_0) - p_U(V_0)D_0(V_0).$$

The value of debt equals

$$D_0(V_0) = \frac{d_0(V_0)}{1 - p_U(V_0)}.$$

The initial (optimal) leverage ratio is given by

$$Lev_0 = \frac{D_0(V_0)}{D_0(V_0) + E(V_0)}.$$

Tinally, the credit spread is given by

$$CS_0 = (C/D_0) - [\tau/(1 - \tau_i)].$$

Step 2 Simulation of the dynamic model

We start with a cross-section of 5000 identical firms. We simulate daily returns assuming 252 trading days per year over a 50 year period. If $V_B^* < V_t < V_U$, then the firm moves to next instant without any action. Each time the firm's asset value reaches the upward refinancing threshold $V_t \geq V_U$, the firm buys back the current debt and issues more debt, and scales up C, V_B, and V_U by γ, where γ is a constant throughout time. If the firm asset value reaches the bankruptcy boundary $V_t \leq V_B^*$, then the firm goes bankrupt.

We generate a time-series of asset values V_t for each firm, and then compute the daily equity value $E(V_t)$. The equity return is measured as a simple return, $r_{i,t}^E = E(V_t)/E(V_{t-1}) - 1$. Finally, we compute monthly and annual IVOL from the daily equity returns.

Figure 1: Example of refinancing

In this figure we show a typical sample path of firm asset value in our simulations, based on the optimal upward-refinancing capital structure strategy in Goldstein et al. (2001). A firm's asset value follows a Geometric Brownian motion. The initial asset value of the firm is V_0= \$100. The asset value (short-dashed green line) follows Geometric Brownian Motion. The long-dashed red line is the upward refinancing threshold of asset value V_U. The solid blue line is the bankruptcy boundary V_B. Period 0 ends either by firm value reaching the bankruptcy boundary $V^*_{B,0}$, at which point the firm declares bankruptcy, or by firm value reaching the upward refinancing threshold $V_{U,0} = \gamma V_0$, at which point the debt is recalled and the firm issues a new, larger amount of debt. γ is the endogenously determined, constant scaling factor. The model is tractable since at the start of each new period the current firm value, the updated bankruptcy and refinancing boundaries and the optimal debt level and coupon are all scaled by γ compared to the start of the previous period. That is, $V_{U,n} = \gamma^n V_{U,0}$, $V_{B,n} = \gamma^n V_{B,0}$, $C_n = \gamma^n V_0$, where n indicates the number of refinancing rounds that have taken place. The simulation runs for 50 years with 12600 days in total.

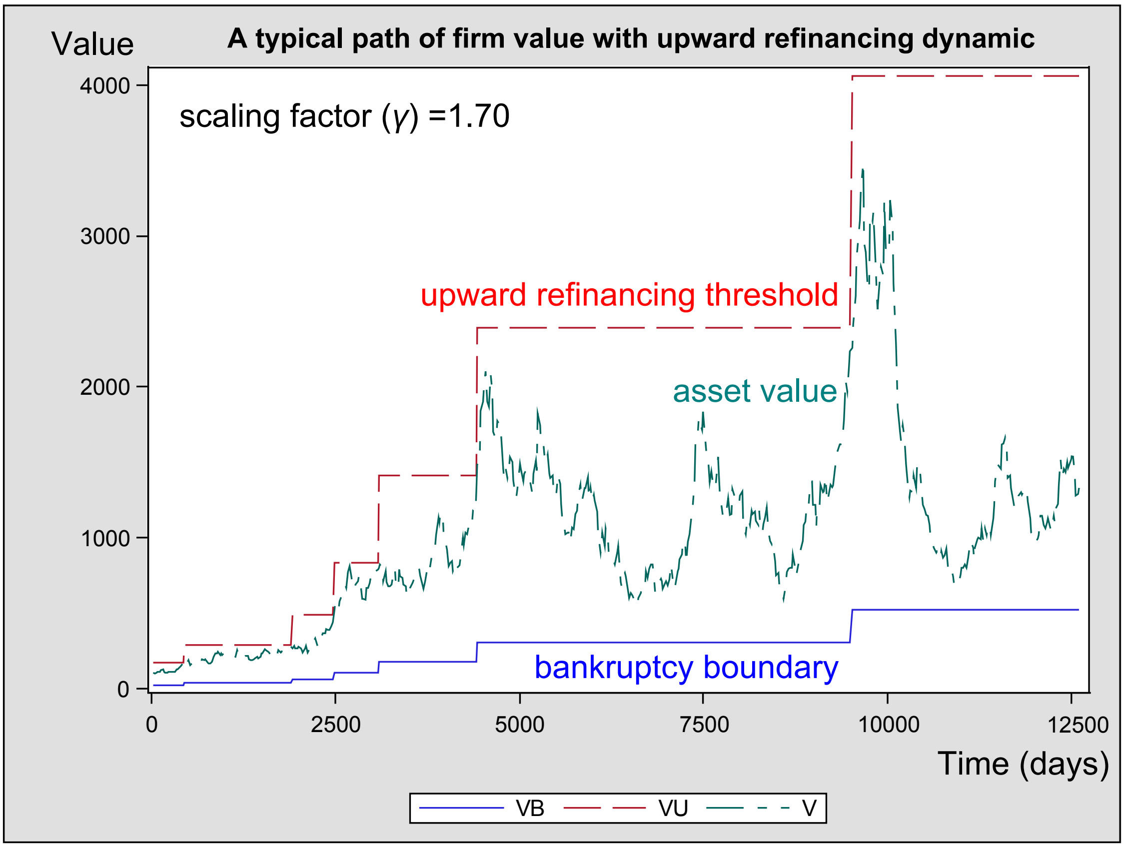


Figure 2: Time-series plots of the main variables

In this figure, we plot the time-series of the main variables. We construct the samples from repeated simulations on the optimal upward refinancing capital structure model in Goldstein et al. (2001). Each simulation run includes 5000 initial firms over 50 years. We compute the market value of debt and equity for each firm in each month. $Leverage = D/(D+E)$. We then compute the cross-sectional average debt, equity, and leverage in each month. In each figure, the solid blue line represents the mean of the corresponding variable in repeated simulations; the two dashed lines represent the 5$^{\text{th}}$ and 95$^{\text{th}}$ percentile values.

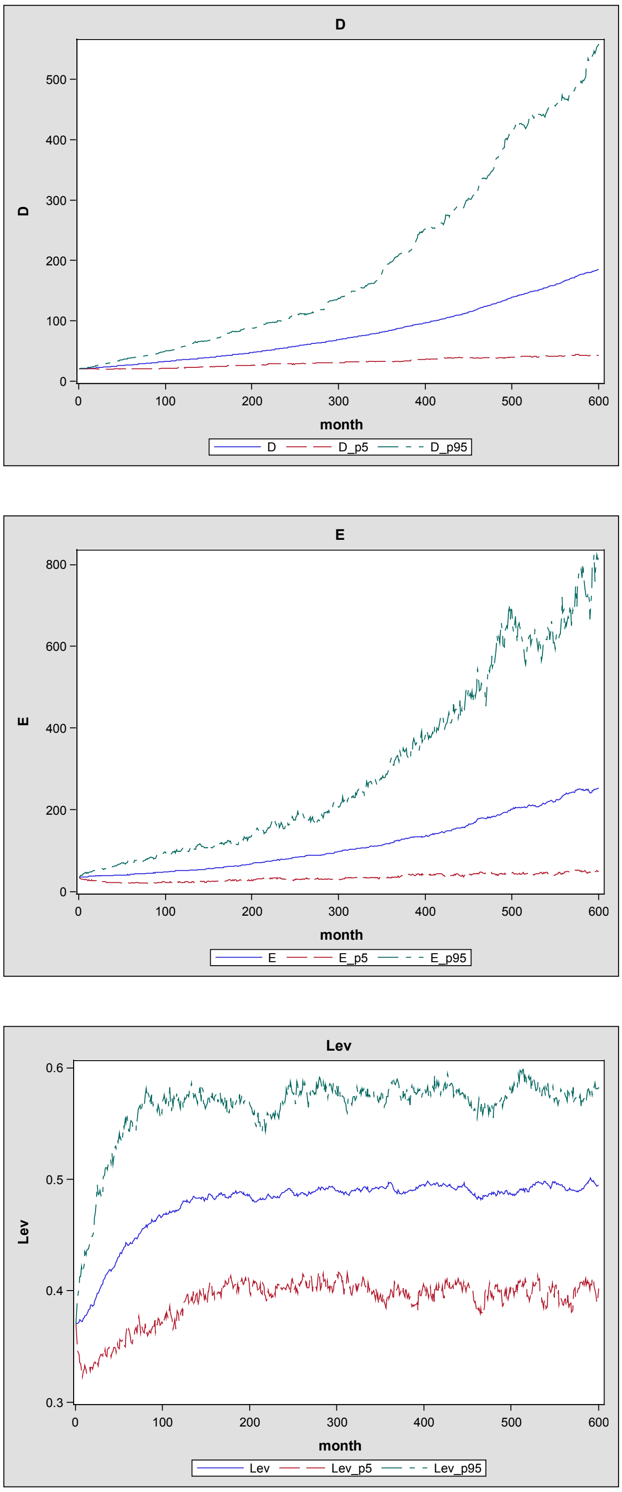
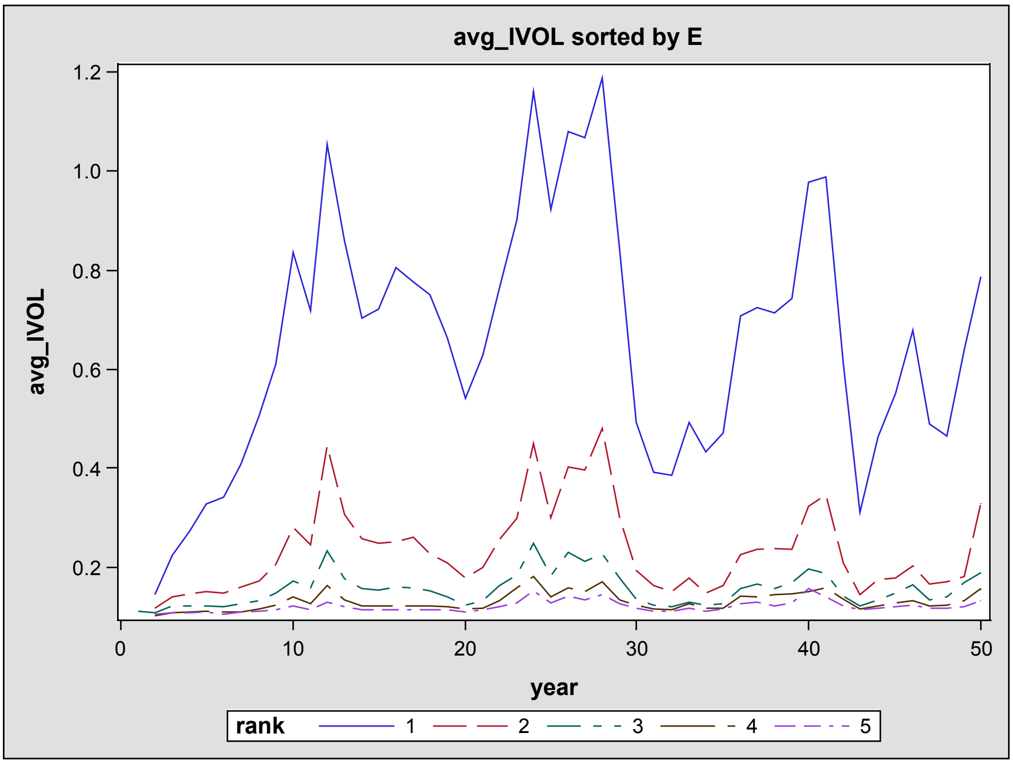


Figure 3: Comovement in portfolios' average IVOL from one simulation run

We compute the annual IVOL as the residual return volatility from the CAPM using daily equity returns. We construct the samples from repeated simulations on the optimal upward refinancing capital structure model in Goldstein et al. (2001). In each simulation run, we sort the stocks by their size (measured as equity value) or by financial leverage into quintiles, and then compute the average annual IVOL for each quintile. Figures (a) and (b) plot the average annual IVOL of the quintiles sorted by size and leverage, respectively, from a typical simulation run.



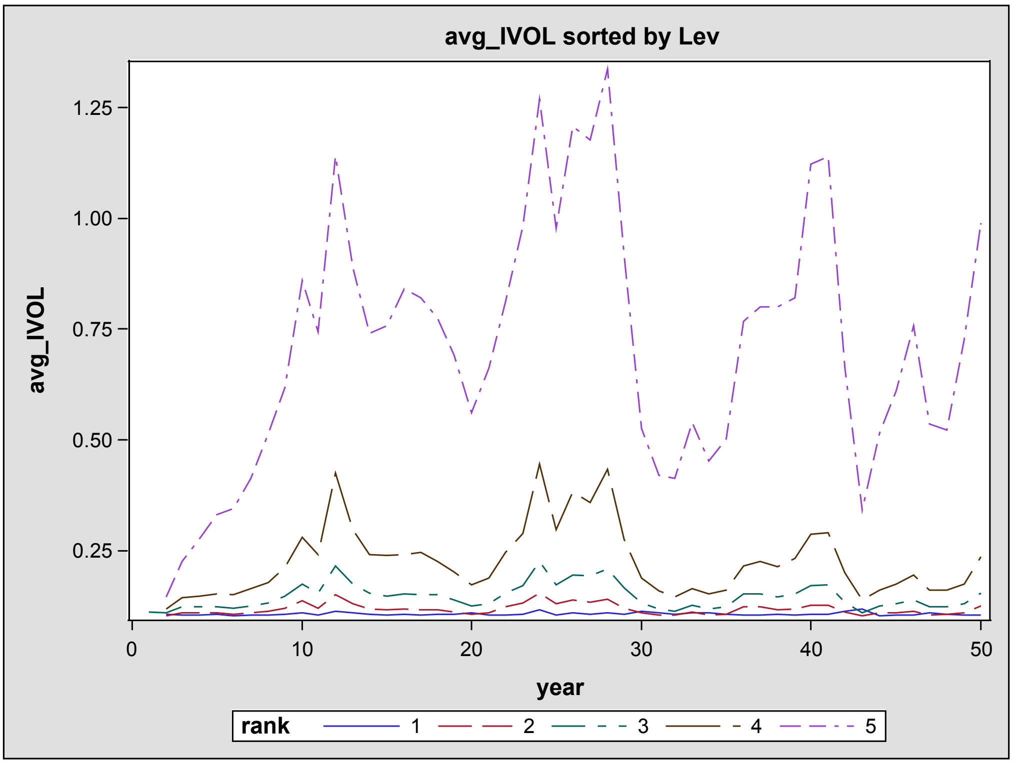


Figure 4: Distribution of average R2

This figure plots the distribution of average R^2 of firm-by-firm single-factor regressions on individual IVOL, in repeated simulations. We construct the samples from repeated simulations on the optimal upward refinancing capital structure model in Goldstein et al. (2001). In each simulation run, we run a time-series regression $IVOL_{i,t} = a_i + b_i CIV_t + \epsilon_{i,t}$ at the firm level, and compute the average R^2 of the regressions on all firms. Annual IVOL for each firm is the volatility of the residual returns from CAPM regressions using daily equity returns. CIV_t is the cross-sectional equally-weighted average IVOL in each year.

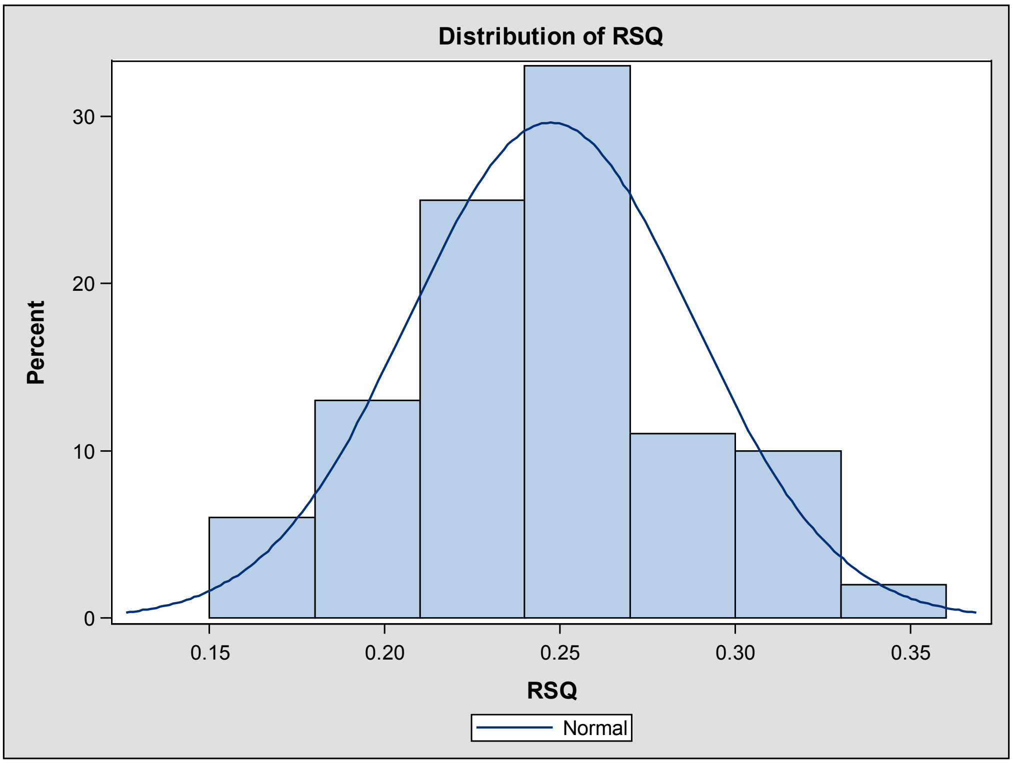


Table 1: Simulation parameter values

This table reports the parameter values used in the simulations.

$V_0 = \$100$	Initial asset value
$\tau_c = 35\%$	Corporate tax rate
$\tau_i = 35\%$	Personal interest income tax rate
$\tau_d = 20\%$	Personal dividend income tax rate
$r_f = 4.5\%$	After tax risk free rate
$\sigma = 0.25$	Asset return volatility
$\alpha = 0.05$	Bankruptcy cost
$\epsilon = 0.5$	Tax shield effective rate when bankruptcy
$q = 0.01$	Restructuring (refinancing) cost rate
$\theta = 0.2$	Sharpe ratio of the common shock dZ
$P/E = 20$	Price-to-earning ratio

Table 2: Summary statistics for the main variables

This table reports the initial value and the means of the main variables. We construct the sample from repeated simulations on the optimal upward refinancing capital structure model in Goldstein et al. (2001). Each simulation run includes 5000 initial firms over 50 years. In the column headed *Initial Value*, we report the optimal initial value of coupon payment (C), bankruptcy boundary (V_B) and refinancing scaling factor (γ). γ is endogenously determined and remains constant. V is the asset value of the firm. D and E are the market values of debt and equity, respectively. $Leverage = D/(D+E)$. $Creditspread = [C/D - r_f(1-\tau_i)] \times 10^4$. r^E is the monthly equity return. $N_s tock$ is the average number of stocks in the cross-section. In each simulation run, we compute the cross-sectional average for each variable and average over the full time-series. Column Mean reports the mean, as well as the 5$^{\text{th}}$ and 95$^{\text{th}}$ percentile values (in brackets) for each variable in repeated simulation runs.

Variables	Initial Value	Mean
γ	1.70	1.70
V	100.00	347.13
		[139.83, 871.46]
C	1.85	7.53
		[3.36, 18.6]
D	20.92	80.43
		[34.93, 199.05]
E	35.61	115.23
		[43.69, 296.63]
$Leverage$	0.37	0.48
		[0.44, 0.51]
$Creditspread$	193.57	260.89
		[237.74, 282.97]
r^E		1.06
		[0.8, 1.29]
N_stock	5000	4928
		[4860.84, 4978.03]

Table 3: IVOL estimated from CAPM and PCA

In this table, we compare the IVOL estimated from CAPM and the IVOL estimated from Principal Component Analysis (PCA). IVOL is estimated as the idiosyncratic variance of the equity returns from CAPM or PCA using daily data. We report the mean of the average IVOL for both measures in repeated simulations. The numbers in brackets are the 5^{th} and 95^{th} percentile values. Row *CORR* shows the full panel correlations between the two measures of IVOL. Panels A and B use the annual and monthly IVOL, respectively. We construct the sample from repeated simulations on the optimal upward refinancing capital structure model in Goldstein et al. (2001).

Panel A Annual		
	$IVOL_{CAPM}$	$IVOL_{PCA}$
MEAN	0.26	0.25
	[0.21, 0.31]	[0.20, 0.30]
STD	0.51	0.48
	[0.38, 0.70]	[0.36, 0.65]
CORR		0.99
		[0.98, 0.99]

Panel B Monthly		
	$IVOL_{CAPM}$	$IVOL_{PCA}$
MEAN	0.24	0.18
	[0.19, 0.28]	[0.15, 0.22]
STD	0.64	0.46
	[0.43, 0.93]	[0.32, 0.66]
CORR		0.96
		[0.94, 0.97]

Table 4: Pairwise correlations

This table reports the average pairwise correlations between the average IVOL of the portfolios sorted by size or leverage. In each simulation run, we sort the firms by size (or by leverage) into quintiles and compute the average IVOL for each quintile in each year, then we compute the pairwise correlations of the average IVOL between the quintiles. We repeat this procedure across repeated simulation runs. Next, we compute the mean, 5^{th} and 95^{th} percentile values (numbers in brackets) of the correlations for each quintile pair over the repeated simulation runs. Panels A and B use annual and monthly data, respectively. We construct the sample from repeated simulations on the optimal upward refinancing capital structure model in Goldstein et al. (2001).

Panel A Annual						
A1 Size	rank	1	2	3	4	5
	1	1.00				
	2	0.95	1.00			
		[0.92, 0.97]				
	3	0.91	0.96	1.00		
		[0.84, 0.96]	[0.92, 0.99]			
	4	0.87	0.91	0.94	1.00	
		[0.75, 0.95]	[0.78, 0.97]	[0.88, 0.98]		
	5	0.81	0.86	0.89	0.93	1.00
		[0.62, 0.94]	[0.65, 0.97]	[0.75, 0.97]	[0.84, 0.98]	
A2 Leverage	rank	1	2	3	4	5
	1	1.00				
	2	0.13	1.00			
		[-0.36, 0.73]				
	3	0.06	0.97	1.00		
		[-0.53, 0.73]	[0.95, 0.99]			
	4	0.10	0.94	0.98	1.00	
		[-0.51, 0.75]	[0.9, 0.97]	[0.97, 0.99]		
	5	0.01	0.85	0.91	0.94	1.00
		[-0.50, 0.59]	[0.71, 0.94]	[0.79, 0.97]	[0.86, 0.98]	

Panel B Monthly

B1 Size	rank	1	2	3	4	5
	1	1.00				
	2	0.96	1.00			
		[0.94, 0.98]				
	3	0.95	0.96	1.00		
		[0.9, 0.97]	[0.92, 0.99]			
	4	0.90	0.89	0.93	1.00	
		[0.79, 0.95]	[0.75, 0.96]	[0.85, 0.97]		
	5	0.73	0.72	0.75	0.81	1.00
		[0.39, 0.91]	[0.34, 0.91]	[0.42, 0.91]	[0.57, 0.93]	

B2 Leverage	rank	1	2	3	4	5
	1	1.00				
	2	-0.17	1.00			
		[-0.42, 0.15]				
	3	-0.14	0.98	1.00		
		[-0.38, 0.17]	[0.96, 0.99]			
	4	-0.11	0.94	0.99	1.00	
		[-0.34, 0.19]	[0.91, 0.97]	[0.97, 0.99]		
	5	-0.11	0.87	0.92	0.95	1.00
		[-0.31, 0.17]	[0.79, 0.94]	[0.86, 0.97]	[0.89, 0.98]	

Table 5: Explaining firm-level IVOL using CIV

This table reports the results of using the common IVOL factor (CIV) to explain the time-series variations in firm-level IVOL. In each simulation run, we run time-series regressions $IVOL_{i,t} = a_i + b_i CIV_t + \epsilon_{i,t}$ for each individual stock over the full sample period and then compute the average R^2 in the cross-section. CIV is measured as the equally-weighted average IVOL. We report the mean of the average R^2 in the repeated simulation runs. The numbers in brackets are the 5^{th} and 95^{th} percentile values of R^2. To compare the relation between the common factor structure in IVOL and financial leverage, we refer to our simulation sample as the *levered* sample. In the corresponding *unlevered* sample, we use the same initial parameter values and the same dynamics of asset value V, but we force the coupon payment C and the bankruptcy boundary V_B equal to zero for all stocks throughout the simulation horizon. Panels A and B use annual and monthly data, respectively. We construct the sample from repeated simulations on the optimal upward refinancing capital structure model in Goldstein et al. (2001).

Panel A Annual			
	R^2	$\hat{a}$	$\hat{b}$
Levered	0.25	-0.14	1.87
	[0.18, 0.31]	[-0.25, -0.01]	[1.35, 2.39]
Unlevered	0.02	-0.05	1.22

Panel B Monthly			
	R^2	$\hat{a}$	$\hat{b}$
Levered	0.13	-0.13	1.88
	[0.09, 0.16]	[-0.21, -0.04]	[1.43, 2.28]
Unlevered	0.00	-0.05	1

Table 6: Determinants of CIV

This table reports the results for linear regressions of the first difference of the common IVOL factor (ΔCIV) on the first differences of market equity return variance ($\Delta\sigma_m^2$), average financial leverage (ΔCS_m), and average credit spread (ΔCS_m). $\Delta CIV_t = \alpha + \beta_1\Delta\sigma^2_{m,t} + \beta_2\Delta Lev_{m,t} + \beta_3\Delta CS_{m,t} + \epsilon_t$. We run this time-series regression at the market level in each simulation run, and report the mean, 5[th] and 95[th] percentile values (numbers in brackets) for the estimated coefficients. The percentages are the percentage of simulation runs in which the estimated coefficient is significant at the 5% level. We construct the sample from repeated simulations on the optimal upward refinancing capital structure model in Goldstein et al. (2001).

Panel A Annual			*Panel B Monthly*		
	(1)	(2)		(1)	(2)
$\Delta\sigma_m^2$	2.45	2.36	$\Delta\sigma_m^2$	0.10	0.10
	[1.44, 3.63]	[1.37, 3.55]		[0.03, 0.22]	[0.03, 0.22]
	99%	99%		84%	85%
ΔLev_m	0.43		ΔLev_m	0.38	
	[0.19, 0.70]			[0.22, 0.56]	
	78%			100%	
ΔCS_m		8.02	ΔCS_m		6.39
		[3.72, 12.39]			[3.83, 9.66]
		89%			100%
$adj.R^2$	0.58	0.61	$adj.R^2$	0.06	0.07
	[0.38, 0.73]	[0.41, 0.75]		[0.03, 0.11]	[0.04, 0.12]
N_{year}	49	49	N_{month}	598	598

Table 7: Returns on portfolios sorted by exposure to CIV

This table reports the cross-sectional return difference of the portfolios sorted by exposure to the common IVOL factor (CIV). We run firm-by-firm time-series regressions $r_{i,t} - r_f = \alpha_{i,t} + \beta_{CIV,i}\Delta CIV_t + \epsilon_{i,t}$ in a 60-month rolling window to estimate $\beta_{CIV,i}$, the exposure to the innovations in CIV. We sort the stocks by β_{CIV} into quintiles in the current month and hold the portfolios in the next month. We construct the sample from repeated simulations on the optimal upward refinancing capital structure model in Goldstein et al. (2001). We report the mean, 5^{th} and 95^{th} percentile values (numbers in brackets) from repeated simulations for each variable in each quintile. N is the number of stocks.

β_{CIV} rank	1-Low	2	3	4	5-High	H-L
β_{CIV}	-3.14	-1.61	-0.95	-0.32	0.77	3.92
	[-3.84, -2.08]	[-2.2, -0.56]	[-1.55, 0.21]	[-0.97, 0.94]	[-0.04, 2.35]	[3.29, 4.71]
$r_{i,t}(EW)$	2.01	0.91	0.77	0.74	1.16	-0.84
	[1.72, 2.25]	[0.69, 1.14]	[0.5, 1.03]	[0.45, 1.01]	[0.89, 1.44]	[-1.18, -0.48]
$r_{i,t}(VW)$	0.59	0.36	0.31	0.3	0.33	-0.25
	[0.29, 0.82]	[0.06, 0.67]	[0.03, 0.59]	[-0.03, 0.58]	[0.01, 0.64]	[-0.37, -0.13]
$\alpha(EW)$	1.46	0.49	0.37	0.34	0.72	-0.74
	[1, 1.93]	[0.34, 0.67]	[0.27, 0.47]	[0.28, 0.43]	[0.5, 0.97]	[-1.11, -0.38]
$\alpha(VW)$	1.04	0.93	0.91	0.92	0.97	-0.07
	[0.86, 1.23]	[0.7, 1.16]	[0.67, 1.15]	[0.65, 1.17]	[0.7, 1.22]	[-0.19, 0.07]
$IVOL$	0.53	0.42	0.4	0.39	0.43	-0.1
$Leverage$	0.57	0.48	0.46	0.46	0.48	-0.09
$Creditspread$	325.69	262.97	249.49	245.54	265.72	-59.96
$Size$	85.67	127.01	140.23	145.81	132.55	46.88
β_{CAPM}	1.59	1.22	1.16	1.15	1.27	-0.32
N	938	939	939	939	939	

Table 8: Returns on portfolios sorted by IVOL

This table reports the cross-sectional return difference of the portfolios sorted by IVOL. We sort the stocks by IVOL into quintiles in the current month and hold the portfolios in the next month. We construct the sample from repeated simulations on the optimal upward refinancing capital structure model in Goldstein et al. (2001). We report the mean, 5th and 95th percentile values (numbers in brackets) from repeated simulations for each variable in each quintile. N is the number of stocks.

$IVOL$ rank	1-Low	2	3	4	5-High	H-L
$IVOL$	0.26	0.32	0.36	0.43	0.72	0.46
	[0.25, 0.27]	[0.3, 0.33]	[0.34, 0.38]	[0.4, 0.47]	[0.61, 0.83]	[0.36, 0.56]
$r_{i,t}(EW)$	0.21	0.41	0.55	0.73	3.2	2.98
	[-0.09, 0.51]	[0.07, 0.74]	[0.16, 0.92]	[0.3, 1.09]	[2.84, 3.55]	[2.36, 3.47]
$r_{i,t}(VW)$	0.03	0.28	0.44	0.62	1.3	1.27
	[-0.25, 0.3]	[-0.01, 0.56]	[0.1, 0.73]	[0.2, 0.97]	[1.06, 1.51]	[1.12, 1.45]
$\alpha(EW)$	-0.11	0.07	0.18	0.30	2.50	2.60
	[-0.13, -0.09]	[0.03, 0.1]	[0.12, 0.22]	[0.24, 0.35]	[1.71, 3.3]	[1.82, 3.41]
$\alpha(VW)$	0.99	0.91	0.83	0.74	1.55	0.56
	[0.77, 1.24]	[0.67, 1.18]	[0.59, 1.09]	[0.52, 0.97]	[1.27, 1.85]	[0.46, 0.66]
β_{CIV}	-0.82	-0.87	-0.94	-1.09	-1.49	-0.68
$Leverage$	0.36	0.39	0.44	0.52	0.68	0.32
$Creditspread$	190.96	207.25	229.81	274.41	403	212.04
$Size$	173.39	149.19	123.95	88.13	42.18	-131.2
β_{CAPM}	0.94	0.98	1.05	1.21	2.01	1.07
N	985	986	986	986	985	

Table 9: Longer-term performance of the portfolios sorted by IVOL

This table reports the monthly returns of portfolios sorted by IVOL. We estimate the monthly IVOL as the residual return volatility from CAPM using daily equity returns. We sort the stocks by IVOL into quintiles in the formation month t, and hold the portfolios in the following three month t+1 to t+3. We construct the sample from repeated simulations on the optimal upward refinancing capital structure model in Goldstein et al. (2001). We report the mean, 5^{th} and 95^{th} percentile values (numbers in brackets) of the portfolio returns from repeated simulations.

Rank	N	t	t+1	t+2	t+3
1-Low	985.24	0.91	0.21	0.25	0.26
		[0.58, 1.25]	[-0.09, 0.51]	[-0.05, 0.56]	[-0.06, 0.57]
2	985.81	0.83	0.41	0.41	0.41
		[0.46, 1.2]	[0.07, 0.74]	[0.07, 0.73]	[0.08, 0.74]
3	985.81	0.77	0.55	0.54	0.54
		[0.41, 1.14]	[0.16, 0.92]	[0.16, 0.88]	[0.17, 0.9]
4	985.81	0.79	0.73	0.75	0.76
		[0.45, 1.11]	[0.3, 1.09]	[0.37, 1.1]	[0.37, 1.1]
5-High	985.43	1.99	3.2	3.06	3.06
		[1.58, 2.31]	[2.84, 3.55]	[2.74, 3.4]	[2.74, 3.41]
High-Low		1.08	2.98	2.81	2.81
		[0.36, 1.61]	[2.36, 3.47]	[2.21, 3.24]	[2.2, 3.26]

Table 10: Full panel correlations

For each simulation run, we compute the full panel correlations between the main variable β_{CIV}, $size$, $IVOL$, $leverage$, $creditspread(CS)$ and r_{t+1} as a proxy for expected equity returns. This table reports the mean, 5$^{\text{th}}$ and 95$^{\text{th}}$ percentile values (numbers in brackets) of the correlations from repeated simulations. We construct the sample from repeated simulations on the optimal upward refinancing capital structure model in Goldstein et al. (2001).

	β_{CIV}	$Size$	$IVOL$	$Leverage$	CS	β_{CAPM}	r_{t+1}
β_{CIV}	1						
$Size$	0.18	1					
	[0.09, 0.26]						
$IVOL$	-0.17	-0.59	1				
	[-0.24, -0.08]	[-0.70, -0.46]					
$Leverage$	-0.21	-0.78	0.73	1			
	[-0.29, -0.11]	[-0.87, -0.67]	[0.67, 0.77]				
CS	-0.21	-0.77	0.81	0.98	1		
	[-0.29, -0.11]	[-0.87, -0.67]	[0.75, 0.84]	[0.97, 0.98]			
β_{CAPM}	-0.14	-0.46	0.61	0.56	0.62	1	
	[-0.19, -0.07]	[-0.54, -0.36]	[0.53, 0.65]	[0.50, 0.60]	[0.56, 0.66]		
r_{t+1}	-0.02	-0.07	0.11	0.07	0.09	0.09	1
	[-0.02, -0.01]	[-0.07, -0.06]	[0.10, 0.12]	[0.06, 0.07]	[0.08, 0.09]	[0.08, 0.10]	

Table 11: Fama-MacBeth regression

This table reports the Fama-MacBeth regression results. The dependent variable is the firm-level equity return. β_{CIV} is the individual firm's exposure to the common IVOL factor CIV. IVOL is the monthly idiosyncratic equity return volatility. We report the mean, 5^{th} and 95^{th} percentile values (numbers in brackets) for the estimated coefficients from repeated simulation runs. The percentages are the percentage of simulation runs in which the estimated coefficient is significant at the 5% level. N is the number of months. We construct the sample from repeated simulations on the optimal upward refinancing capital structure model in Goldstein et al. (2001).

	(1)	(2)	(3)
β_{CIV}	-0.32	0.13	-0.01
	[-0.50, -0.17]	[0.05, 0.25]	[-0.07, 0.05]
	99%	93%	14%
$IVOL$		10.38	
		[9.54, 11.58]	
		100%	
$Leverage$			9.89
			[9.18, 10.66]
			100%
$adj.R^2$	0.01	0.05	0.04
N	540	540	540

Table 12: Correlations between financial leverage and credit spread

In the first row, we calculate the market average financial leverage and the market average credit spread in each year (or month), and compute the correlations between the two. Similarly, in the second row, we compute the correlation between the changes in market average financial leverage and the changes in market average credit spread. This table reports the mean, 5^{th} and 95^{th} percentile values (numbers in brackets) of the correlations from repeated simulations. We construct the sample from repeated simulations of the optimal upward refinancing capital structure model in Goldstein et al. (2001).

	Annual	Monthly
CORR (CS, Leverage)	0.997	0.997
	[0.995, 0.998]	[0.996, 0.998]
CORR (CS, Leverage)	0.994	0.990
	[0.990, 0.997]	[0.985, 0.994]

Table 13: Persistence of IVOL ranking in the high IVOL quintile

We focus on the stocks sorted into the top IVOL quintile in the formation month t, and further divide the quintile in the subsequent month (t+1) based on whether they remain in the top IVOL quintile, and repeat the sorting in month $t + 2$ and $t + 3$. "H" refers to the stocks that remain in the top IVOL quintile, "L" refers to stocks that drop out of the top IVOL quintile. For example, groups HHH and HHL are constructed by splitting the stocks in group HH based on whether or not they remain in the top IVOL quintile by the end of month $t + 2$. This table reports the mean, 5^{th} and 95^{th} percentile values (numbers in brackets) of the number of stocks and the returns for each group from repeated simulations. N is the number of stocks. We construct the sample from repeated simulations of the optimal upward refinancing capital structure model in Goldstein et al. (2001).

rank	N	t	t+1	t+2	t+3
HHH	507.9				4.32
	[439.29, 539.99]				[3.91, 4.76]
HHL	120.57				1.01
	[116.93, 127.94]				[0.48, 1.45]
HH	627.63			3.88	3.84
	[567, 657.28]			[3.5, 4.27]	[3.48, 4.25]
H	985.43	1.99	3.2	3.06	3.06
	[971.98, 995.43]	[1.58, 2.31]	[2.84, 3.55]	[2.74, 3.4]	[2.74, 3.41]
HL	359.9			0.87	1.1
	[315.35, 426.42]			[0.4, 1.27]	[0.8, 1.4]
HLH	105.26				1.61
	[96.24, 116.19]				[1.35, 1.89]
HLL	255.17				0.78
	[219.3, 310.82]				[0.38, 1.16]